The Returns on Human Capital: Good News on Wall Street is Bad News on Main Street

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We use a standard single-agent model to conduct a simple consumption growth accounting exercise. Consumption growth is driven by news about current and expected future returns on the market portfolio. We impute the residual of consumption growth innovations that cannot be attributed to either news about financial asset returns or future labor income growth to news about expected future returns on human wealth, and we back out the implied human wealth and market return process. Innovations in current and future human wealth returns are negatively correlated with innovations in current and future financial asset returns, regardless of the elasticity of intertemporal substitution. (JEL G12, G14, G33)

1. Introduction

Starting with the seminal work by Lucas (1978) and Breeden (1979), much of the work in dynamic asset pricing asks whether observed aggregate consumption growth can deliver financial returns like the ones we observe in the data [Grossman and Shiller (1981) and Hansen and Singleton (1983)]. Mehra and Prescott (1985) point out that a very high degree of risk aversion is needed to reconcile a high equity premium with a low covariance between consumption growth and returns. Kandel and Stambaugh (1990) extend this analysis to the conditional moments of returns and consumption growth, and they reach the same conclusion. We turn this question on its head: starting from observed returns on financial assets, what restrictions does the standard single-agent model impose on...
the joint distribution of the market returns and aggregate consumption growth?

Following Roll (1977) critique, the literature has recognized the importance of including human wealth returns as part of the market return [Shiller (1995), Campbell (1996), and Jagannathan and Wang (1996)]. Only the cash flow component of human wealth returns is observed, not the discount rate component. This paper uses observed aggregate consumption to identify the discount rate component in human wealth returns. A standard single-agent model puts tight restrictions on the joint distribution of market returns and aggregate consumption, and we exploit these restrictions to conduct a basic consumption growth accounting exercise: we impute that part of the consumption innovations that cannot be attributed to news about current or future financial returns to the returns on human wealth.

We find that (1) good news about current returns in financial markets is bad news about current returns in labor markets, regardless of the elasticity of intertemporal substitution (EIS), and (2) the implied total market return is negatively correlated with the returns on financial wealth if the EIS is smaller than one. The negative correlation between financial and human wealth returns is driven by a cash flow component and a discount rate component. First, good news about future labor income growth is bad news for the future growth rate of payouts to securities holders. This cash flow correlation is a feature of the data. Second, positive innovations to future risk premia on financial wealth tend to coincide with negative innovations to expected future returns on human wealth. This discount rate effect is what comes out of our consumption growth accounting exercise.

The negative discount rate correlation for these two assets is not surprising. Santos and Veronesi (2004) were the first to point out the effect of this composition on risk premia in a two-sector model. Consider a simple example of a two-tree Lucas endowment model with i.i.d dividend growth and log preferences. When the dividend share of the first tree increases, its expected return must go up to induce investors to hold it despite its larger share. Because the overall price-dividend ratio stays constant, the expected return on the second tree has to decrease [see Cochrane et al. (2004)].

Innovations in aggregate consumption are determined by news about current returns and by news about future expected returns on the market portfolio. The effect of news about future market returns on consumption depends only on how willing the representative agent is to substitute consumption intertemporally, and not on her risk preferences [Campbell (1993)]. If the portfolio only includes financial wealth, the model-implied consumption innovations are radically different from those in the data. The agent’s consumption innovations are at least five times too volatile relative
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to US data, and the implied correlation of her consumption innovations with news about stock returns is also four times too high. Even when the \( EIS \) is zero, there is just not enough mean reversion in financial asset returns to reconcile the moments of consumption and returns. We call this the consumption correlation and volatility puzzle. It is another manifestation of the equity volatility puzzle of LeRoy and Porter (1981) and Shiller (1981). These two moments of aggregate consumption growth are also at the heart of Mehra and Prescott’s (1985) equity premium puzzle. However, the volatility and correlation puzzles only depend on the agent’s willingness to transfer consumption between different periods in response to news about future returns, while the equity premium puzzle only depends on the agent’s aversion to consumption bets. In a model with only financial wealth, there is no value of the \( EIS \) that closes the gap between the model and the data, but large values definitely make matters worse. We want to investigate how much of this failure can be attributed to market return mismeasurement.

To do so, we explicitly introduce human wealth in our single agent’s portfolio, following the example of Campbell (1996), Shiller (1995), and Jagannathan and Wang (1996). In the first step, we show that a model in which the expected returns on human wealth and financial wealth are perfectly correlated, like Campbell’s (1996), cannot come close to matching the consumption moments in the data. The fact that the consumption dynamics implied by the intertemporal capital asset pricing model (ICAPM) are inconsistent with actual aggregate consumption dynamics might explain why the ICAPM that substitutes out consumption does well in explaining the cross-section of asset returns [Campbell (1993, 1996, 2004)], whereas the ICAPM that uses consumption data does not [Yogo (2006) and Epstein and Zin (1991)]. Models in which the expected return on human wealth is constant, like Shiller’s (1993), or in which the expected return on human wealth is perfectly correlated with expected labor income growth, like Jagannathan and Wang’s (1996), do better, but these still over-predict the volatility of consumption innovations and their correlation with financial returns. We conclude that these models incorrectly account for the risk in human wealth. Instead, we back out the human wealth returns directly from aggregate consumption data.

While Campbell’s work aimed to substitute consumption out of the asset pricing equations, our aim is to obtain better measures of market risk by forcing the market return to be consistent with the moments of aggregate consumption. The resulting equity premia are very different. In all the benchmark models that we discuss, the hedging component of the risk premium [Merton (1973)] is negative, because stocks are less risky in the long run. This is obvious in the simplest case with only financial wealth: the mean reversion in stock returns offsets much of the contemporaneous stock market risk. As a result, these models cannot generate large risk
premia. In the consumption-consistent model, the hedging risk premium is always positive, stocks are riskier and risk premia larger.

1.1 Related literature and discussion

While there is a huge amount of literature on the risk-return trade-off in financial markets, the role of risk is usually ignored when economists model human capital investment decisions. Palacios-Huerta (2001) is the first to focus on this trade-off in labor markets; he uses individual labor-income-based measures of human capital returns. We use the information in aggregate consumption innovations instead to learn about the aggregate human wealth returns. In a related work, Restoy and Weill (1998) treat total wealth as an unobservable and show how to recover its returns from consumption. Their focus is on re-deriving the ICAPM. Our paper takes their insight to the data, albeit in a very different way, and shows that it matters. The consumption-consistent market return is very different from what we usually think of as the market return, and this has important repercussions for asset pricing. Bansal and Yaron (2004) deliver a consumption and dividend process that can match expected returns on financial wealth by imputing a key role to long-run consumption risk. Instead, we back out a human wealth return process that implies the right aggregate consumption behavior. Vissing-Jorgensen and Attanasio (2003) also use Campbell’s (1996) framework to estimate the EIS and the coefficient of risk aversion using household-level data. They conclude that the EIS of stockholders is likely to be above one, but they do not match the model-implied consumption volatility and correlation moments with those in the data. Our paper adds these two consumption moments to the picture. Our work is also related to that of Santos and Veronesi (2004). They set up a two-sector model, a labor income and a capital income generating sector; assets are priced off a conditional CAPM in which the labor income share is the conditioning variable. Interestingly, Boyd et al. (2005) show that, on average, good news about unemployment implies lower stock returns. Similar results are obtained for a wide range of macro-economic announcements by Andersen et al. (2005). We infer from aggregate consumption that bad news for stock returns is good news for the rest of the economy.

Our work also has clear portfolio implications. US household portfolios are biased towards US securities. If financial and human wealth returns are negatively correlated, human wealth provides a good hedge against domestic asset return movements, rationalizing a long position in home assets. Relying on co-integration analysis, Julliard (2003) reaches the

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1 Palacios-Huerta measures returns on human capital as the proportional increase in earnings per year from the last year of schooling. He does not take into account the effect of revisions on future labor income and discount rates; see also Palacios-Huerta (2003).
same conclusion as we do, contradicting earlier results by Baxter and Jermann (1997), who conclude that introducing the labor income risk unambiguously worsens the international diversification puzzle because long-run labor income and financial income are positively correlated. This makes human wealth look like stocks. Relying on the same positive correlation between long-run labor income growth and stock returns, Benzoni et al. (2005) manage to explain the hump-shaped life-cycle pattern of stock market participation. Our model suggests that this positive correlation has counter-factual implications for the implied consumption of these investors.\(^2\) We argue that the cross-equation restrictions on consumption may help to identify the nature of long-run human capital risk.

We start by briefly reviewing the Campbell framework in Section 2. In Section 3, we describe the data we use, we explain how we operationalize the model, and we estimate the model subject to a co-integration restriction on consumption, financial wealth, and labor income. In Section 4 we describe the consumption correlation and volatility puzzle by showing that all four benchmark models fail to deliver plausible aggregate consumption dynamics. Then, we reverse-engineer human wealth returns to match aggregate consumption data. Finally, we study the equity premium generated by the various models. In Section 5, we entertain potential alternative explanations of our findings. The last section concludes. A separate appendix with additional derivations and tables is available on the authors’ web sites.\(^3\)

2. Environment

We adopt the environment of Campbell (1993) and consider a single-agent decision problem.

2.1 Preferences

The agent ranks consumption streams \(\{C_t\}\) using the following utility index \(U_t\), which is defined recursively:

\[
U_t = \left( 1 - \beta \right)^{(1 - \gamma)/\theta} + \beta \left( E_t U_{t+1}^{1 - \gamma} \right)^{\theta/(1 - \gamma)},
\]

where \(\gamma\) is the coefficient of relative risk aversion and \(\sigma\) is the EIS. Finally, \(\theta\) is defined as \(\frac{1 - \gamma}{1 - \gamma/\sigma}\). In the case of separable utility, the EIS equals the

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\(^2\) In all the papers cited above, the evidence on the correlation between cash flow and discount rate risk of human and financial wealth returns relies mostly on co-integration analysis. Such co-integration tests are known to have low power. For example, Hansen et al. (2005) show flat likelihood plots for the co-integration coefficient between consumption and earnings.

\(^3\) http://www.econ.ucla.edu/hlustig/ and http://pages.stern.nyu.edu/~svnieuwe/.
inverse of the coefficient of risk aversion and $\theta$ is one. These preferences, due to Epstein and Zin (1989), impute a concern to agents about the timing of the resolution of uncertainty. This plays a potentially important role in understanding risk premia [Bansal and Yaron (2004)]. Distinguishing between the coefficient of risk aversion and the inverse of the EIS will prove important. The restrictions on the joint distribution of financial wealth returns, human wealth returns, and consumption will depend only on the EIS, not on the coefficient of risk aversion.

2.2 Trading assets

All wealth, including human wealth, is tradable. We adopt Campbell’s notation: $W_t$ denotes the representative agent’s total wealth at the start of period $t$, and $R^m_{t+1}$ is the gross return on wealth invested from $t$ to $t+1$. This representative agent’s budget constraint is

$$W_{t+1} = R^m_{t+1} (W_t - C_t).$$

(1)

Our single agent takes the returns on the market $\{R^m_t\}$ as given, and decides how much to consume. Instead of imposing market clearing and forcing the agent to consume aggregate dividends and labor income, we simply let the agent choose the optimal aggregate consumption process, taking the market return process $\{R^m_t\}$ as given.

2.3 The joint distribution of consumption and asset returns

Campbell (1993) linearizes the budget constraint and uses the Euler equation to obtain an expression for consumption innovations as a function of innovations to current and future expected returns. Lowercase letters denote logs.

First, Campbell linearizes the budget constraint around the mean log consumption–wealth ratio $c - w$. If the consumption–wealth ratio is stationary, in the sense that $\lim_{j \to \infty} \rho^j (c_{t+j} - w_{t+j}) = 0$, the approximation implies that

$$c_{t+1} - E_t c_{t+1} = (E_{t+1} - E_t) \sum_{j=0}^{\infty} \rho^j r^m_{t+1+j} - (E_{t+1} - E_t) \sum_{j=1}^{\infty} \rho^j \Delta c_{t+1+j},$$

(2)

where $r^m = \log(1 + R^m)$ and $\rho$ is defined as $1 - \exp(c - w)$.\(^4\) Equation (2) says that, in the long run, the returns on the market portfolio are

\(^4\) Campbell (1993) shows that this approximation is accurate for values of the EIS between 0 and 4.
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completely driven by aggregate consumption growth: only cash flows matter and not discount rates.

Second, Campbell assumes consumption and returns are conditionally homoscedastic and jointly log normal, and substitutes the consumption Euler equation

$$E_t \Delta c_{t+1} = \mu_m + \sigma E_t r^{m}_{t+1},$$

where $\mu_m$ is a constant that includes the variance and covariance terms for consumption and market return innovations, back into the budget constraint (2), to obtain an expression with only returns on the right hand side:

$$c_{t+1} - E_t c_{t+1} = r_{t+1}^{m} - E_t r^{m}_{t+1} + (1 - \sigma)(E_t r_{t+1} - E_t) \sum_{j=1}^{\infty} \rho^j r_{t+1+j}^{m}.$$  

Campbell shows that this agent incurs relatively small welfare losses from using this linear consumption rule. We will use this linear version of the model as our actual model.

Innovations to the representative agent’s consumption are determined by (1) the unexpected part of this period’s market return and (2) the innovation to expected future market returns. There is a one-for-one relation between current return and consumption innovations, regardless of the EIS, but the relation between consumption innovations and innovations to expected future returns depends on the EIS. If the agent has log utility over deterministic consumption streams and $\sigma$ is one, the consumption innovations exactly equal the unanticipated return in this period. If $\sigma$ is larger than one, the agent lowers her consumption to take advantage of higher expected future returns, whereas if $\sigma$ is smaller than one, she chooses to increase her consumption because the income effect dominates the substitution effect. As $\sigma$ approaches zero, the current consumption innovations equal the long-run market return innovations, as is apparent from comparing the linearized budget constraint in (2) and the consumption equation in (4).

The consumption function in (4) puts tight restrictions on the joint distribution of aggregate consumption innovations and total wealth return innovations. Our aim is to study the properties of aggregate consumption implied by this restriction. More specifically, we are interested in two moments of the consumption innovations: (1) the variance of consumption.

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5 Hansen et al. (1991) and appendix A.4 show that, if the state vector that describes the dynamics of returns and dividends follows a linear autoregressive process, this long-run restriction cannot be satisfied for all innovations. This is true in the simplest case of one asset (e.g., only financial wealth) or in the case of multiple assets and constant wealth shares. Partly to circumvent this problem, we will introduce time-varying wealth shares in the analysis. This destroys the linearity in the relation between consumption and return innovations.

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innovations and (2) the correlation of consumption innovations with financial return innovations.

3. Data and Model Implementation

This section discusses the measurement of financial asset returns (Section 3.1), the computation of the innovation series that feed into the consumption function (section 3.2), and the relevant moments of these innovations in US post-war data (Section 3.3). Finally, we explain how we measure the market return (Section 3.4).

3.1 Measuring financial asset returns

We use two measures of financial asset returns. The first is the return on the value-weighted Center for Research in Security Prices (CRSP) stock market portfolio:

\[ R_{t+1}^a = \frac{P_{t+1} + D_{t+1}}{P_t}, \]

where \( D_t \) is the dividend in period \( t \) and \( P_t \) is the ex-dividend price.\(^6\) The full line in Figure 1 shows the log dividend-price ratio \( dp_{t}^a \). We follow the literature on repurchases [Fama and French (2001) and Grullon and Michaely, 2002], and adjust the dividend yield for total repurchases of equity to ensure that it remains stationary.\(^7\) The resulting series is the dotted line in Figure 1. The dividend-price ratio adjusted for repurchases is similar to the unadjusted series until 1980, and consistently higher afterwards.

Our second measure of financial asset returns takes a broader perspective by including corporate debt and private companies. We value a claim to US nonfinancial, nonfarm corporations and compute the total payouts to the owners of this claim. This “firm value” is measured as the market value of equity plus the market value of all financial liabilities minus the market value of financial assets. The payout measure includes all corporate payouts to securities holders, both stock holders and bond holders.\(^8\) The dashed line in Figure 1 shows the ratio of payouts to securities holders to the market value of firms. Over the last two decades, the dividend

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\(^6\) Our benchmark case is annual data. When we consider quarterly data instead, we define the log dividend-price ratio as \( dp_{t}^q = \log \left( \frac{25D_t + 25D_{t-1} + 25D_{t-2} + 25D_{t-3}}{P_t} \right) \) to remove the seasonal component in dividends.

\(^7\) Lettau and Van Nieuwerburgh (2006) show that the null of no structural breaks cannot be rejected against the alternative of one or two breaks for this series. The repurchase data are from Boudoukh et al. (2004) and start in 1971. Adjusting for net repurchases instead of total repurchases does not change the results.

\(^8\) The computation of firm value returns is based on Hall (2001). The data to construct our measure of returns on firm value were obtained from the Federal Flow of Funds. We calculate the value of all securities as the sum of financial liabilities (144990005), the market value of equity (1031640030) less financial assets (144909005), adjusted for the difference between market and book for bonds. We correct for changes in the market value of outstanding bonds by applying the Dow Jones Corporate Bond Index to the level of outstanding corporate bonds at the end of the previous year. The flow of payouts is measured as dividends (10612005) plus the interest paid on debt (from National Income and Product Accounts (NIPA) table on Gross Product of nonfinancial, corporate business) less the increase in net financial liabilities (10419005), which includes issues of equity (103164003).
yield for the firm value measure has been much higher than the dividend yield on stocks, consistent with the findings of Hall, (2001). This broader measure of financial wealth is our benchmark, but we also report the results using stock market wealth because the latter is more commonly used.

Table 1 reports the moments of the CRSP stock returns and firm value returns and the corresponding dividend growth rates. The quarterly returns and dividend growth rates are annualized. The correlation between the two returns series is high: .95 for annual data and .89 for quarterly data. The firm value returns have a 1.3% higher mean and a 3% lower volatility than stock returns. There is virtually no serial correlation in either of these returns. However, the dividend data are very different. The correlation of dividend growth rates for these two measures is only .40 in annual data and .22 in quarterly data. In addition, the standard deviation of payout growth is much higher, 17% in annual data, compared to only 11% for the narrow dividends. The higher cash flow volatility is consistent with the findings of Larrain and Yogo (2005).

3.2 Computing innovations
Following Campbell (1996), we use a VAR to represent the law of motion for the state vector. We make two technical contributions to this methodology. First, we exploit additional restrictions imposed by the co-integration of consumption, wealth and labor income, following Lettau and Ludvigson (2001). Second, we allow for time-varying wealth shares (see Section 3.4.4). Both innovations strengthen our results.

The $N \times 1$ state vector $z_t$ is given by $z'_t = (\Delta a_t \Delta y_t \ dpa^*_t \ reltb_t \ ysp_t \ s_l \ \Delta c_l)$, which includes the change in log real financial wealth ($\Delta a_t$), real labor income growth ($\Delta y_t$), three return predictors—the log dividend–price ratio on financial assets ($dpa^*_t$), the relative T-bill return ($reltb_t$), and the yield spread ($ysp_t$)—as well as the labor income share...
Table 1
Moments of returns and dividend growth

<table>
<thead>
<tr>
<th>Moments</th>
<th>Panel A: Firm value</th>
<th>Panel B: Stock market</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Quarterly</td>
<td>Annual</td>
</tr>
<tr>
<td>( E[r^a] )</td>
<td>.0893</td>
<td>.0897</td>
</tr>
<tr>
<td>( \text{Std}[r^a] )</td>
<td>.1418</td>
<td>.1411</td>
</tr>
<tr>
<td>( \text{Corr}[r^a, r^a_{t-1}] )</td>
<td>.0340</td>
<td>.0599</td>
</tr>
<tr>
<td>( \text{Corr}[\Delta_a, \Delta_S, \Delta_C] )</td>
<td>.8907</td>
<td>.9527</td>
</tr>
</tbody>
</table>

The data span 1947–2004. Panel A uses the broad firm value measure. Panel B uses the narrow stock market wealth measure. The upper panel lists the moments of log real returns. The lower panel lists the moments of log real dividend growth. The deflation uses the personal income deflator; the same deflator used to deflate all series in the paper. The quarterly returns and dividend growth rates are annualized. In each panel, row 1 reports the sample mean. Row 2 reports the sample standard deviation. Row 3 reports the first order serial correlation. Row 4 reports the contemporaneous correlation between the stock measure and the firm value measure.

\( (s_t) \) and real consumption growth (\( \Delta C_t \)). Our measure of consumption is real non-durables and services consumption excluding housing services. Our measure of financial wealth (\( a_t \)) is either the CRSP stock market capitalization (corresponding to our measure of stock returns) or the total market value of nonfarm nonfinancial business (corresponding to firm value returns). The change in wealth (\( \Delta a_t \)) is constructed from the returns (\( r^a_{t+1} \)) as follows:

\[
\Delta a_t = r^a_t + k + \left( 1 - \frac{1}{\rho} \right) \Delta a_{t-1},
\]

where \( k \) is a linearization constant. We include the change in financial wealth in the state vector instead of the return because we impose co-integration on consumption, wealth, and labor income. All state variables are demeaned.

3.2.0.1 Co-integration. Lettau and Ludvigson (2001) find evidence for a long-run relation between consumption, financial wealth, and labor income, and call the deviation from this co-integration relationship \( cay_t \):

\[
cay_t = \lambda C_t - (1 - \bar{v})a_t - \bar{y}t,
\]

where \( \lambda = 1.0395 \) is the ratio of log total to log nondurable and services consumption. Using data from 1947 to 2004 on (\( c, a, y \)), we estimate the
human wealth share to be $\nu = 0.7761$ using the broad firm value measure for $a$, and we obtain $\nu = 0.7923$ using the stock market wealth measure.\textsuperscript{9}

The co-integration among consumption, financial wealth, and human wealth imposes restrictions on the state transition matrix $A$ and on the errors $\epsilon$. The dynamics of the state vector are described by a Vector Error Correction Model (VECM):

$$z_{t+1} = Az_t + \Gamma cay_t + \epsilon_{t+1}, \tag{7}$$

with innovation covariance matrix $E[\epsilon \epsilon'] = \Sigma$. The dimensions of $\Sigma$ and $A$ are $N \times N$, the dimensions of $\epsilon$ and $z$ are $N \times T$. In addition, the dimensions of $\Gamma$ and $cay$ are $N \times 1$ and $1 \times T$. Following Cochrane (1994), we can rewrite this VECM in VAR form

$$\begin{bmatrix} z_{t+1} \\ cay_{t+1} \end{bmatrix} = \begin{bmatrix} \tilde{A} & \tilde{\Gamma} \\ \tilde{\Gamma} & \tilde{\epsilon} \end{bmatrix} \begin{bmatrix} z_t \\ cay_t \end{bmatrix} + \begin{bmatrix} \epsilon_{t+1} \\ \tilde{\epsilon}_{t+1} \end{bmatrix} \tag{8}$$

where $\tilde{A}, \tilde{\Gamma}$ and $\tilde{\epsilon}$ are given by

$$\tilde{A} = \lambda A_7 - (1 - \nu)e_1 - \nu e_2,$$

$$\tilde{\Gamma} = 1 + \lambda \Gamma_7 - (1 - \nu)\Gamma_1 - \nu \Gamma_2,$$

$$\tilde{\epsilon} = (\lambda e_7 - (1 - \nu)e_1 - \nu e_2) \epsilon,$$

$A_i$ denotes the $i^{th}$ row of the matrix $A$, and $e_k$ is the $k^{th}$ column of an identity matrix of the same dimension as $A$. The co-integration relationship imposes restrictions on the last equation of the $cay$-augmented VAR. We estimate $A$, $\Gamma$, and $\Sigma$ from the VECM in (7) and construct the augmented VAR according to Equation (8).\textsuperscript{10}

### 3.2.0.2 Extracting innovations

Once the VAR has been estimated, we can extract the cash flow and discount rate innovations that drive consumption growth innovations:

$$c_t = c_t - E_{t-1}[c_t] = \Delta c_t - E_{t-1}[\Delta c_t] = e_t^y \epsilon_t, \tag{9}$$

Table 2 defines the notation and shows how to recover each expression from the VAR innovations. The $CF$ label denotes news about cash flows, while the $DR$ label denotes news about discount rates (returns). The superscript $y$ denotes human wealth, $a$ denotes financial wealth,

\textsuperscript{9} See Appendix A.2 for details on the model with co-integration and the estimation procedure. Our results are robust to lower values for $\nu$.

\textsuperscript{10} The VAR companion matrix coefficient estimates are reported in Table 3 of the separate appendix. The entries have the expected sign. Lagged financial wealth growth has a positive effect on next period’s consumption and income growth. The dividend-price ratio is a marginally significant return predictor (not reported in table).
Table 2
Notation: discount rate and cash flow innovations

<table>
<thead>
<tr>
<th>Label</th>
<th>Definition</th>
<th>Expression</th>
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</thead>
<tbody>
<tr>
<td>(DR^a_t)</td>
<td>(\epsilon'_t)</td>
<td>(\epsilon'_t)</td>
</tr>
<tr>
<td>(DR^\infty_a)</td>
<td>((E_t - E_{t-1})\sum_{j=1}^{\infty} \rho^j \Delta a_{t+j})</td>
<td>(\epsilon'_t(1-\rho)(I-\rho A)^{-1} \epsilon'_t)</td>
</tr>
<tr>
<td>(CF^\infty_t)</td>
<td>(\Delta y_t - E_{t-1} \Delta y_t)</td>
<td>(\epsilon'_t(1-\rho)(I-\rho A)^{-1} \epsilon'_t)</td>
</tr>
<tr>
<td>(CF^a_t)</td>
<td>(\Delta a_t - E_{t-1} \Delta a_t)</td>
<td>(\epsilon'_t + \epsilon'_3 \epsilon'_t)</td>
</tr>
<tr>
<td>(CF^\infty_t)</td>
<td>((E_t - E_{t-1})\sum_{j=0}^{\infty} \rho^j \Delta a_{t+j})</td>
<td>(\epsilon'_t(1-\rho)(I-\rho A)^{-1} \epsilon'_t)</td>
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</table>

News about cash flows is denoted \(CF\) and news about discount rates is denoted \(DR\). The superscript \(y\) denotes human wealth, while \(a\) denotes financial wealth. The subscript \(t\) denotes current innovations, \(\infty\) denotes future innovations and \(t, \infty\) denotes current and future innovations. In line 1, note that \(DR^a_t = \Delta a_t - E_{t-1} \Delta a_t\) from Equation (5). In line 2, note that \(DR^\infty_a = \Delta a_t - E_{t-1} \Delta a_t\) from Equation (5) and further algebraic manipulation.

In line 6, note that \(CF^\infty_t = DR^\infty_a + DR^2_t\).

and \(m\) denotes the market, or total wealth. The subscript \(t\) denotes current innovations, \(\infty\) denotes future innovations and \(t, \infty\) denotes current and future innovations. In the next section, we identify the human wealth discount rate innovations \(DR^y_\infty\) and \(DR^y_t\) from the consumption innovations, but first, we highlight some surprising facts about the cash flow innovations in the data.

3.3 Stylized facts
We use our VAR estimates to compute second moments of cash flow and discount rate innovations. The stylized facts about discount rates are well-documented (at least for stock returns), less so for the stylized facts about cash flows.

3.3.0.3 Discount rate news. The first panel in Table 3 summarizes the moments for discount rate innovations. The left panel uses the firm value returns as the measure of financial asset returns; the right panel uses stock market returns. The first column reports results for quarterly data (1947–2004); the second column reports the results for annual data (1947–2004). For stock returns, we also report the moments for a longer sample (1930–2004). Throughout, we will focus on annual 1947–2004 data as our benchmark. The numbers in parentheses are the standard errors generated by means of an i.i.d. bootstrap; the numbers in brackets are generated from a “wild” bootstrap procedure that accounts for conditional heteroscedasticity (see appendix A.1 for details).
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Table 3
Cash flow and discount rate news

<table>
<thead>
<tr>
<th>Moments</th>
<th>Panel A: Firm value</th>
<th>Panel B: Stock market</th>
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<tbody>
<tr>
<td></td>
<td>Quarterly</td>
<td>Annual</td>
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### Panel I: Discount rate news

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<tbody>
<tr>
<td>Std(DR(t))</td>
<td>0.139</td>
<td>0.135</td>
<td>0.160</td>
<td>0.156</td>
<td>0.176</td>
</tr>
<tr>
<td>(0.009)</td>
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<td>[0.016]</td>
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<tr>
<td>Std(DR(a))</td>
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</tr>
<tr>
<td>(0.035)</td>
<td>(0.039)</td>
<td>(0.015)</td>
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<td>[0.045]</td>
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<td>[0.024]</td>
<td>[0.027]</td>
</tr>
<tr>
<td>Corr(DR(t), DR(a))</td>
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<td>-0.972</td>
<td>-0.965</td>
<td>-0.892</td>
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<td>(0.058)</td>
<td>(0.086)</td>
<td>(0.025)</td>
<td>(0.035)</td>
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<td>[0.082]</td>
<td>[0.021]</td>
<td>[0.028]</td>
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</tr>
<tr>
<td>Std(c)</td>
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<td>0.111</td>
<td>0.088</td>
<td>0.17</td>
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<tr>
<td>Corr(c, DR(t))</td>
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<td>[0.069]</td>
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</table>

### Panel II: Cash flow news

<p>| | | | | | |</p>
<table>
<thead>
<tr>
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<tbody>
<tr>
<td>Std(C(F_{t,\infty}))</td>
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<td>0.034</td>
<td>0.033</td>
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<td>(0.007)</td>
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<td>(0.013)</td>
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<tr>
<td></td>
<td>[0.011]</td>
<td>[0.010]</td>
<td>[0.013]</td>
<td>[0.013]</td>
<td>[0.011]</td>
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<tr>
<td>Corr(C(F_{t,\infty}), DR(t))</td>
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<td>(0.223)</td>
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<td>(0.292)</td>
<td>(0.261)</td>
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<td>[0.245]</td>
<td>[0.290]</td>
<td>[0.282]</td>
<td>[0.233]</td>
</tr>
<tr>
<td>Corr(C(F_{t,\infty}), DR(a))</td>
<td>-0.633</td>
<td>-0.689</td>
<td>-0.336</td>
<td>-0.348</td>
<td>-0.297</td>
</tr>
<tr>
<td>(0.240)</td>
<td>(0.244)</td>
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<td>(0.269)</td>
<td>(0.229)</td>
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<tr>
<td></td>
<td>[0.245]</td>
<td>[0.261]</td>
<td>[0.272]</td>
<td>[0.283]</td>
<td>[0.244]</td>
</tr>
<tr>
<td>Corr(C(F_{t,\infty}), C(F_{t,\infty}))</td>
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<td>-0.371</td>
<td>-0.346</td>
<td>-0.205</td>
<td>-0.294</td>
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<tr>
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<td>(0.306)</td>
<td>(0.362)</td>
<td>(0.235)</td>
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<tr>
<td></td>
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<td>[0.334]</td>
<td>[0.298]</td>
<td>[0.369]</td>
<td>[0.258]</td>
</tr>
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</table>

The table reports annualized standard deviations (\(Std\)) and correlations (\(Corr\)) in the data. Panel A uses the returns on firm value. Panel B uses the return on the value-weighted CRSP stock index. The first column in each panel reports results for quarterly data (1947–2004). The second column reports results for annual data (1947–2004). The last column reports results for a longer sample of annual data (1930–2004). The subscript \(a\) denotes financial wealth, \(y\) denotes human wealth. \(CF\) denotes cash flow news and \(DR\) denotes discount rate news. \(c\) denotes innovations to non-durable and services consumption. The standard errors in ( ) are generated by bootstrapping with replacement from the VAR residuals. The standard errors in [ ] are generated by a wild bootstrap (robust to heteroscedasticity).
• Firm value return innovations are about 15 times more volatile than consumption innovations. The standard deviation of news about current financial returns ($\text{Std}(D_{R_a}^t)$) is 13.5% for firm value returns and 15.6% per annum for stock returns; the same number for consumption is 0.8% per annum ($\text{Std}(c)$).

• Consumption innovations and return innovations are only weakly correlated: the correlation ($\text{Corr}(c, D_{R_a}^t)$) is .21 for firm value returns and .23 for stock returns. On the basis of bootstrapped standard errors (in parentheses and brackets), the null hypothesis that this correlation is zero cannot be rejected at the 1% level.

• News about future financial returns is also volatile. In annualized terms, the standard deviation is 14.3% for firm value and 12.8% for stock returns ($\text{Std}(D_{R_a}^\infty)$).

• Current return innovations are negatively correlated with news about future expected returns ($\text{Corr}(D_{R_a}^t, D_{R_a}^\infty) < 0$). There is strong (multivariate) mean reversion in the returns on firm value ($-0.86$) and even more in stock returns ($-0.96$).

The first two facts are at the heart of the consumption volatility and correlation puzzle. The quarterly data provide a similar picture. In the long annual sample (last column), consumption innovations are more volatile, but still 10 times less volatile than stock returns.

### 3.3.0.4 Cash flow news

The second panel in Table 3 summarizes the moments of cash flow innovations. Again focusing on annual data, three facts stand out.

• For both firm value and stock market data, news about current and future dividend growth and labor income growth are negatively correlated ($\text{Corr}(C_{F_y}^t, C_{F_y}^\infty, C_{F_a}^t, C_{F_a}^\infty) < 0$).

• Periods with good news about current financial asset returns tend to be periods with good news about current and future labor income growth ($\text{Corr}(C_{F_i}^t, D_{R_a}^t) > 0$).\(^{11}\)

• Periods with good news about future financial asset returns tend to be periods with bad news about current and future labor income growth ($\text{Corr}(C_{F_i}^\infty, D_{R_a}^\infty) < 0$).

Good cash flow news for securities holders (stock and bond holders) is bad cash flow news for workers. When we look at the narrow stock market wealth measure, the (long-run) correlation between current and future innovations in labor income and financial income is also

---

\(^{11}\) The correlation between financial discount rate innovations and human wealth cash flow innovations is more precisely measured using firm value returns. When using stock market returns, these correlations have the same sign but are not significantly different from zero.
negative. This correlation is hard to measure because it involves an infinite sum. For annual data this correlation is not significantly negative, but it is at quarterly frequencies. This cash flow channel is the first important component of our results. It may be even stronger in other countries: Bottazzi et al. (1996) document strong negative contemporaneous correlation between wage and profit rates in a large cross-section of developed countries.

### 3.4 Measuring the market return

The market portfolio includes a claim to the entire aggregate labor income stream. The total market return can be decomposed into the return on financial assets $R^a$ and returns on human capital $R^y$. Using log returns, we have

$$r_t^m = (1 - v_t^{-1}) r_t^a + v_t^{-1} r_t^y,$$

where $v_t$ is the ratio of human wealth to total wealth. The innovation to the return on human capital equals the innovation to the expected present discounted value of labor income less the innovation to the present discounted value of future returns. The Campbell (1991) decomposition implies that

$$r_t^y - E_t^{-1} r_t^y = (E_t - E_{t-1}) \sum_{j=0}^{\infty} \rho_j \Delta y_{t+j} - (E_t - E_{t-1}) \sum_{j=1}^{\infty} \rho^j r_{t+j}^y,$$

or equivalently, $DR_t^y = CF_{t,\infty}^y - DR_{\infty}^y$. A windfall in human wealth returns is driven by higher expected labor income (cash flow) growth or by lower expected risk premia (discount rates) on human wealth. To an econometrician, the human wealth discount rate news time series $\{DR_{t,i}\}$ is unobserved, and, therefore, so is the time series of current innovations to human wealth returns $\{DR_t^y\}$. We deal with this in two different ways. First, we adopt the approach taken by others before us. Models I–IV specify a linear process for the human wealth discount rate $E_t[r_t^y]$. We show that these existing models produce consumption that is inconsistent with the data. Second, we find the discount rate process on human wealth that implies consumption moments consistent with the data (Model V).

#### 3.4.1 Model I: financial Wealth

We start by abstracting from nonfinancial wealth, by setting $v_t = 0, \forall t$, and we compare the model-implied consumption innovation behavior to aggregate US data. We call the model financial wealth Model I. This is a natural starting point, because standard business cycle models imply that the returns on human and other assets are highly or even perfectly correlated [e.g., Baxter and Jermann (1997)]. Likewise, in finance, it is standard practice to use the
stock market return $r_t^m$ as a measure of the market return $r_t^m$ [Black (1987) and Stambaugh (1982)].

3.4.2 Models II–IV: three benchmark models of human wealth returns.

Next, we introduce human wealth by setting up three different models that have been used in the literature. The three benchmark models differ only in the $N \times 1$ vector $C$, which measures how the innovations to the expected human wealth returns relate to the state vector:

$$E_t[r_{t+1}^y] = C'z_t.$$  

In Model II, the model of Campbell (1996), expected human wealth returns are assumed to equal expected financial asset returns:

$$E_t[r_{t+1}^y] = E_t[r_{t+1}^a], \forall t.$$  

Because of Equation (5) and the fact that $\Delta a$ is the first element of the VAR, we have $C' = \frac{1}{\rho} \left( \epsilon_1' \rho + (1 - \rho) \epsilon_1' \right)$. In Model III, the model of Shiller (1995), the discount rate on human capital is constant:

$$E_t[r_{t+1}^y] = 0, \forall t,$$  

and, therefore, $C' = 0$. In Model IV, the model of Jagannathan and Wang (1996), the innovation to human wealth return equals the innovation to the labor income growth rate. The underlying assumptions are that (i) the discount rate on human capital is constant, implying that the second term in Equation (11) is zero, and (ii) labor income growth is unpredictable, so that the first term in Equation (11) is $\Delta y_{t+1} = E_t \Delta y_{t+1}$. The corresponding vector is $C' = \epsilon_2'A$.

Each of these models for the expected returns on human wealth, or equivalently a $C$ vector, implies a process for $\{DR_y^\infty\}$, the innovations to expected future returns on human wealth:

$$DR_y^\infty = C' \rho (I - \rho A)^{-1} \varepsilon_t,$$  

and a process for $\{DR_y^y\}$, the current innovation to the return on human wealth:

$$DR_y^y = CF^y_{t,\infty} - DR_y^\infty = \epsilon_2'\rho (I - \rho A)^{-1} \varepsilon_t - C' \rho (I - \rho A)^{-1} \varepsilon_t.$$  

For example, in the JW model, Equation (13) implies that $C'$ needs to equal $\epsilon_2'\rho$ for $DR_{t+1}^y$ to equal $\Delta y_{t+1} - E_t \Delta y_{t+1} = \epsilon_2'\varepsilon_{t+1}$.

3.4.3 Model V: reverse engineering human wealth returns. Finally, in Model V, we choose the vector $C$, which relates the expected return on human wealth to the state vector, $E_t[r_{t+1}^y] = C'z_t$, to minimize the distance between the model-implied consumption volatility and correlation moments and the same moments in the data.\(^{12}\) This vector then delivers

\(^{12}\) We use a nonlinear least squares algorithm to find the vector $C$ that minimizes the distance between the two model-implied and the two observed consumption moments. Because the moments are highly nonlinear in the $N \times 1$ vector $C$, we cannot rule out that the $C$ vector is not uniquely identified.
human wealth return processes \( DR^w_x \) and \( DR^y_i \) from Equations (12) and (13) and ultimately consumption innovations as shown later.

3.4.4 Time-varying wealth shares. Campbell (1996) keeps the human wealth share constant at the labor income share: \( v_t = \pi = \tau \). We extend his approach to deal with time-variation in the portfolio shares, a necessary extension because we allow the expected returns on both assets to differ. We first derive a linear expression for the human wealth share \( v_t(z_t) \), and then we show how to compute consumption innovations.

3.4.4.1 Computing the human wealth share. When the expected return on human wealth is a linear function of the state (with loading vector \( C \)), the log dividend-price ratio on human wealth \( dpy_t \) is also linear in the state. In particular, the demeaned log dividend-price ratio on human wealth is a linear function of the state \( z \) with a \( N \times 1 \) loading vector \( B \):

\[
dpy_t - E[dpy_t] = \sum_{j=1}^{\infty} \rho^j (r_{t+j} - \Delta y_{t+j}) = \rho(C' - e'2)(I - \rho A)^{-1}z_t = B'z_t.
\]

The demeaned log dividend-price ratio on financial assets is also a linear function of the state, because it is simply the third element in the VAR: \( dpa_t - E[dpa_t] = e'3z_t \). The price-dividend ratio for the market is the wealth-consumption ratio; it is a weighted average of the price-dividend ratio for human wealth and for financial wealth:

\[
\frac{W}{C} = \frac{P^w}{Y} + \frac{P^a}{D}.
\]

Finally, the human wealth to total wealth ratio is given by

\[
v_t = \frac{P^w}{C} = \frac{e^{-dpy_t} s_t}{e^{-dpy_t} s_t + e^{-dpa_t} (1 - s_t)} = \frac{1}{1 + e^{s_t}}, \tag{15}
\]

which is a logistic function of \( x_t = dpy_t - dpa_t + \log \left( \frac{1+s_t}{1-s_t} \right) \), where \( dpa_t = -\log \left( \frac{P^a}{P} \right) \). We recall that \( s \) denotes the labor income share \( s_t = Y_t / C_t \) with mean \( \tau \). When \( dpa_t = dpy_t \), the human wealth share equals the labor income share \( v_t = s_t \). In general, \( v_t \) moves around not only when the labor income share changes, but also when the difference between the log dividend-price ratios on human and financial wealth changes. It is increasing in the former, and decreasing in the latter. In Section A.3 of the appendix, we derive a linear approximation to the logistic function in (15). The demeaned human wealth share \( \tilde{v}_t \equiv v_t - \pi = D'z_t \) is a linear function

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of the state, with loading vector $D$ given by

$$D \equiv e_6 - \overline{\sigma}(1 - \overline{\sigma})B + \overline{\sigma}(1 - \overline{\sigma})e_3. \quad (16)$$

### 3.4.4.2 Consumption innovations.

When wealth shares are time-varying, the agent considers the effect of (future) changes in the portfolio share of each asset when adjusting consumption to news about returns. Combining Equations (4), (10), and (11), the expression for consumption innovations becomes

$$(c)_t = (1 - \nu_{t-1})DR^a_t + \nu_t C F_{t, \infty}^y - \nu_{t-1}DR^y_\infty$$

$$+ (1 - \sigma)(E_t - E_{t-1}) \sum_{j=1}^{\infty} \rho^j (1 - \nu_{t-1+j})r^a_{t+j}$$

$$+ (1 - \sigma)(E_t - E_{t-1}) \sum_{j=1}^{\infty} \rho^j \nu_{t-1+j}r^y_{t+j}. \quad (17)$$

Future returns are now weighted by future, random, portfolio shares. To deal with this complication, we define the news about weighted, future financial asset returns and human wealth returns as follows:

$$DR^w,a_t \equiv (E_t - E_{t-1}) \sum_{j=1}^{\infty} \rho^j \tilde{\nu}_{t-1+j}r^a_{t+j}$$

and

$$DR^w,y_t \equiv (E_t - E_{t-1}) \sum_{j=1}^{\infty} \rho^j \tilde{\nu}_{t-1+j}r^y_{t+j}. \quad (18)$$

Using these definitions and $\tilde{\nu}_t \equiv \nu_t - \overline{\nu}$, the expression for consumption innovations reduces to

$$(c)_t = (1 - \overline{\nu} - \tilde{\nu}_{t-1})DR^a_t + (\tilde{\nu}_{t-1} + \overline{\nu})C F_{t, \infty}^y - (\tilde{\nu}_{t-1} + \sigma\overline{\nu})DR^y_\infty$$

$$+ (1 - \sigma)(1 - \overline{\nu})DR^a_\infty - (1 - \sigma)(DR^w,a_t - DR^w,y_t). \quad (17)$$

When the human wealth share is constant ($\nu_{t-1} = \overline{\nu}$ or $\tilde{\nu}_{t-1} = 0$), we obtain the simpler expression

$$(c)_t = (1 - \overline{\nu})DR^a_t + \overline{\nu}C F_{t, \infty}^y - \sigma\overline{\nu}DR^y_\infty + (1 - \sigma)(1 - \overline{\nu})DR^a_\infty. \quad (18)$$

Consumption responds one-for-one to news about current asset returns, weighted with the capital income share, and to news about discounted current and future labor income growth, weighted with the labor income share, regardless of the $EIS$. The response to news about future asset returns is governed by $1 - \sigma$. The response to news about future human wealth returns is governed by $-\sigma$. This reflects the direct effect of future
human wealth risk premia on consumption and the indirect effect on the current human wealth returns (see Equation 11). In the log case ($\sigma = 1$), variation in future returns or in future human wealth shares has no bearing on consumption innovations today. In any other case, our single agent responds to news about future returns weighted by the portfolio shares. We compute the innovations $DR^{h,a}_t$ and $DR^{h,y}_t$ using value function iteration (see appendix A.3).

4. Consumption Correlation and Volatility Puzzle

In this section, we study the model-implied consumption innovation of Models I–V. Only the consumption innovations in Model V match those in the data.

4.1 Model I: financial wealth

We analyze the moments of the consumption innovations implied by Model I by setting the human wealth share to zero in (17). We then feed the actual innovations to financial asset returns and news about future returns into the linearized consumption function. This procedure delivers a time series for the model-implied consumption innovations. We focus on two moments of these consumption innovations: the standard deviation

$$\text{Std}(c) = \sqrt{\left(\text{Var}(DR^{a}_t) + (1-\sigma)^2\text{Var}(DR^{a}_\infty)\right) + 2(1-\sigma)\text{Cov}(DR^{a}_t, DR^{a}_\infty)},$$

(19)

and the correlation of consumption innovations with innovations to current financial asset returns:

$$\text{Corr}(c, DR^{a}_t) = \frac{\text{Var}(DR^{a}_t) + (1-\sigma)\text{Cov}(DR^{a}_t, DR^{a}_\infty)}{\text{Std}(c)\text{Std}(DR^{a}_t)},$$

(20)

where $\text{Std}(c)$ is given by Equation (19).

Table 3 shows that, in the data, the standard deviation of consumption innovations is only 0.8% per annum, compared to 13.5% (15.6%) per annum for firm (stock) return innovations, and the correlation with return innovations is .207 (.237). Model I fails to match either moment for all values of EIS. Figure 2 plots the standard deviation of the model-implied consumption innovations in the top panel and their correlation with current return innovations in the bottom panel. In both panels, the EIS on the x-axis is varied from 0 to 1.5. The dotted line plots the results for firm value returns; the full line plots the results for stock returns.

First, we focus on the results obtained using the broader measure. In the log case ($\sigma = 1$), consumption responds one-for-one to current return innovations. The standard deviation of consumption innovations equals the standard deviation of news about current financial returns, which is 13.5% per annum (see Equation 19), and the correlation of consumption
innovations with financial asset return innovations is 1 (see Equation 20). As the \( EIS \) decreases below 1, consumption also absorbs part of the volatility of shocks to future asset returns \( \text{Var}(DR_a) \). The effect on the variance of consumption innovations can be mitigated by the mean reversion in returns \( (\text{Cov}(DR_a, DR_a^\infty) < 0) \). If \( \sigma < 1 \), a negative covariance of current and future return innovations also lowers the covariance of consumption with current return innovations: the agent adjusts her consumption by less in response to a positive surprise if the same news lowers her expectation about future asset returns. Indeed, Figure 2 illustrates that the mean reversion in returns helps lower the implied volatility and correlation of consumption innovations somewhat, but not nearly enough.\(^{13}\)

On the other hand, mean reversion in returns actually increases the volatility of consumption if the \( EIS \) exceeds one. That is why the standard deviation of consumption news increases in the top panel as \( EIS \)

---

\(^{13}\) When we use stock returns instead of firm value returns, Model I comes a little closer to matching the volatility (full line in Figure 2). The standard deviation of model-implied consumption innovations drops to 5%. However, even for low \( EIS \), the correlation between consumption and stock market return innovations never drops below 0.7.
increases.\textsuperscript{14} We refer to these two failures of \textit{Model I} as the consumption volatility and the consumption correlation puzzle. These are both tied to the lack of a large financial wealth effect on aggregate consumption. The next section adds human wealth to the single agent’s portfolio.

4.2 Adding human wealth

Table 4 summarizes the moments of consumption and human capital return news for annual 1947–2004 data. Columns 1–4 in each panel report the properties of human wealth returns and consumption for \textit{Model II} \cite{Campbell1996}, \textit{Model III} \cite{Shiller1995}, \textit{Model IV} \cite{Jagannathan1996} and \textit{Model V}, the reverse-engineered model. The average human wealth share $\nu$ is .776 for firm value and .792 for stock market data. We use our benchmark calibration with $EIS$ set to .28, a compromise between the low macro-estimates of \cite{Hall1988}—close to zero—and the consensus estimate of .5 from micro data \cite[see][]{Browning2000}. The left panel reports the results using firm value returns; the right panel is for stock returns. As was the case for \textit{Model I}, \textit{Models II, III, & IV} cannot match the low volatility of consumption innovations and their low correlation with financial asset return innovations. Only \textit{Model V} matches the consumption moments in the data.

4.2.1 Model V: consumption growth accounting. \textit{Model V} treats the expected returns component of human wealth return innovations as a residual. We reverse-engineer the human wealth return process that most closely matches the moments of consumption. The fourth column in each panel of Table 4 shows that this procedure is successful. \textit{Model V}'s consumption moments are very close to those in the data: the volatility is 1.2%, within one standard deviation of the data, and the correlation with financial asset returns is .207, exactly as in the data. For stock market data, the model-implied volatility is only 1% and the correlation is matched exactly.

What are the properties of human wealth returns that enable us to match the two consumption moments? The main novel feature of \textit{Model V} is the large negative correlation it delivers between innovations to human and financial wealth returns: \textit{Corr}(\textit{DR}^y_t, \textit{DR}^a_t) < 0. This plays a key role in matching the consumption moments. To understand what drives this negative correlation, it is helpful to break it down, using our expression for the human wealth return innovation $\textit{DR}^y_t$, into news about cash flows

\textsuperscript{14} There is little evidence for an $EIS$ in excess of one. \cite{Browning2000} conduct an extensive survey of the consumption literature that estimates the $EIS$ using household data; they conclude the consensus estimate is less than one, around .5 for food consumption.
Table 4
Human wealth discount rate news

<table>
<thead>
<tr>
<th>Model</th>
<th>II</th>
<th>III</th>
<th>IV</th>
<th>V</th>
<th>II</th>
<th>III</th>
<th>IV</th>
<th>V</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Panel A: Firm value returns</td>
<td>Panel B: Stock returns</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td></td>
<td>Future human wealth discount rate news</td>
<td>Future human wealth discount rate news</td>
<td></td>
<td></td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Std(DR')</td>
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<td>0.014</td>
<td>0.108</td>
<td>0.128</td>
<td>0.023</td>
<td>0.130</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.031)</td>
<td>(0.007)</td>
<td>(0.033)</td>
<td>(0.019)</td>
<td>(0.012)</td>
<td>(0.046)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Corr(DR', DR'y)</td>
<td>-0.362</td>
<td>0.897</td>
<td>0.843</td>
<td>-0.965</td>
<td>0.373</td>
<td>0.591</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.085)</td>
<td>(0.282)</td>
<td>(0.175)</td>
<td>(0.037)</td>
<td>(0.311)</td>
<td>(0.241)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Corr(DR'y, DR'y)</td>
<td>0.370</td>
<td>0.025</td>
<td>0.322</td>
<td>-0.478</td>
<td>0.136</td>
<td>0.234</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.321)</td>
<td>(0.434)</td>
<td>(0.310)</td>
<td>(0.277)</td>
<td>(0.403)</td>
<td>(0.354)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Corr(DR'y, CF'y)</td>
<td>1.000</td>
<td>0.830</td>
<td>-0.627</td>
<td>1.000</td>
<td>0.404</td>
<td>-0.633</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.000)</td>
<td>(0.370)</td>
<td>(0.283)</td>
<td>(0.000)</td>
<td>(0.311)</td>
<td>(0.253)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Current human wealth discount rate news</td>
<td>Current human wealth discount rate news</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Std(DR')</td>
<td>0.161</td>
<td>0.024</td>
<td>0.019</td>
<td>0.092</td>
<td>0.143</td>
<td>0.033</td>
<td>0.017</td>
<td>0.102</td>
</tr>
<tr>
<td></td>
<td>(0.035)</td>
<td>(0.007)</td>
<td>(0.002)</td>
<td>(0.03)</td>
<td>(0.004)</td>
<td>(0.02)</td>
<td>(0.001)</td>
<td>(0.041)</td>
</tr>
<tr>
<td>Corr(DR', DR'y)</td>
<td>0.847</td>
<td>0.531</td>
<td>-0.846</td>
<td>0.916</td>
<td>0.225</td>
<td>-0.72</td>
<td>-0.682</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.086)</td>
<td>(0.222)</td>
<td>(0.178)</td>
<td>(0.068)</td>
<td>(0.264)</td>
<td>(0.137)</td>
<td>(0.222)</td>
<td></td>
</tr>
<tr>
<td>Corr(DR'y, DR'y)</td>
<td>-0.994</td>
<td>-0.689</td>
<td>-0.285</td>
<td>-0.976</td>
<td>-0.348</td>
<td>-0.120</td>
<td>-0.697</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.005)</td>
<td>(0.244)</td>
<td>(0.293)</td>
<td>(0.021)</td>
<td>(0.269)</td>
<td>(0.152)</td>
<td>(0.240)</td>
<td></td>
</tr>
<tr>
<td>Consumption news</td>
<td>Consumption news</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Std(c)</td>
<td>0.050</td>
<td>0.028</td>
<td>0.026</td>
<td>0.012</td>
<td>0.054</td>
<td>0.034</td>
<td>0.029</td>
<td>0.010</td>
</tr>
<tr>
<td></td>
<td>(0.008)</td>
<td>(0.005)</td>
<td>(0.004)</td>
<td>(0.006)</td>
<td>(0.010)</td>
<td>(0.010)</td>
<td>(0.007)</td>
<td>(0.006)</td>
</tr>
<tr>
<td>Corr(c, DR'y)</td>
<td>0.936</td>
<td>0.823</td>
<td>0.795</td>
<td>0.864</td>
<td>0.636</td>
<td>0.664</td>
<td>0.237</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.040)</td>
<td>(0.170)</td>
<td>(0.169)</td>
<td>(0.089)</td>
<td>(0.190)</td>
<td>(0.164)</td>
<td>(0.115)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.043)</td>
<td>(0.190)</td>
<td>(0.191)</td>
<td>(0.102)</td>
<td>(0.229)</td>
<td>(0.199)</td>
<td>(0.152)</td>
<td></td>
</tr>
</tbody>
</table>

Panel A uses firm value returns. Panel B uses stock returns. All results are for the full sample 1947–2004 (annual data). In each panel, the first column is Model II, with \( C' = \frac{\nu}{\rho} (1 + \rho \nu' c) \). The second column is Model III with \( C' = 0 \), and the third column is Model IV with \( C' = \nu \frac{\rho}{\nu} \). The last column is Model V with \( C' \) chosen to minimize the distance between the model-implied and actual consumption news standard deviation and correlation. Computations are done for \( \nu = 0.7761 \) in panel A and \( \nu = 0.7923 \) in panel B, and \( \rho = 0.28 \). The standard errors in ( ) are generated by bootstrapping with replacement from the VAR residuals. The standard errors in [ ] are generated by a wild bootstrap (robust to heteroscedasticity).
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on human wealth \((C F^y_{t, \infty})\) and news about discount rates \((D R^y_{\infty})\):

\[
\text{Cov}[D R^y_t, D R^a_t] = \text{Cov}[C F^y_{t, \infty}, D R^a_t] - \text{Cov}[D R^y_{\infty}, D R^a_t] < 0
\]

\(@ cash flows \)

\(@ discount rates \)

In the data, periods with good news about current financial asset returns tend to also be periods with good news about current and future labor income growth: the first term is positive \(\text{Corr}(C F^y_{t, \infty}, D R^a_t) > 0\), as we learned from Table 3. To prevent the agent from going on a consumption binge, good news for current financial asset returns needs to coincide with higher future risk premia on human wealth. That is exactly what Model \(V\) accomplishes: \(\text{Corr}(D R^y_{\infty}, D R^a_t) >> 0\). If we dig a bit deeper, we can further decompose each term into two underlying components:

\[
\text{Cov}[D R^y_t, D R^a_t] = \text{Cov}[C F^y_{t, \infty}, D R^a_t] - \text{Cov}[D R^y_{\infty}, D R^a_t] + \text{Cov}[D R^y_{\infty}, D R^a_t]
\]

\(@ cash flows \)

\(@ discount rates \)

4.2.1.1 Human wealth cash flows. Decomposing the first term, we uncover two opposing effects. Good news about current and future cash flows on human wealth coincides with bad news about current and future cash flows for financial assets, but also with lower future risk premia on financial assets. Both are features of the data, not of our identification procedure. The former, the cash flow channel, helps to keep the volatility of consumption in check, but the latter works in the opposite direction.

4.2.1.2 Human wealth discount rates. To overcome this last effect, Model \(V\) chooses the discount rates on human wealth \(D R^y_{\infty}\) that is high when expected future dividend growth is high \(\text{Corr}(D R^y_{\infty}, C F^y_{t, \infty}) > 0\) and future risk premia on financial assets are low \(\text{Corr}(D R^y_{\infty}, D R^a_{\infty}) < 0\). This is the discount rate channel. So, good news on Wall Street is bad news on Main street, both for cash flows (first term) and for discount rates (second term).

4.2.1.3 Simple case: constant wealth shares. To develop some intuition for the discount rate channel, we abstract from time-variation in the wealth shares. The actual consumption innovations in the data can be formed from the VAR residuals as in (9). Plugging these consumption innovations into the household’s linear policy rule (18), we can simply back out the
implied news in future human capital returns:

\[ DR_y^\infty = \frac{1}{\sigma\nu} \left[ (1 - \nu)DR_a^\infty + \nu CF_y^\infty + (1 - \sigma)(1 - \nu)DR_a^\infty - (c) \right]. \tag{21} \]

From this time series \( \{DR_y^\infty\} \) and the data on labor income growth, we recover the innovations to current human wealth returns using this identity 

\[ DR_y^t \equiv CF_y^t,\infty - DR_y^\infty: \]

\[ DR_y^t = \left( 1 - \frac{1}{\sigma} \right) CF_y^t,\infty - \frac{1}{\sigma\nu} \left[ (1 - \nu)DR_a^t \right. \\
+ \left. (1 - \sigma)(1 - \nu)DR_a^\infty - (c) \right]. \]

Any windfall gain in financial markets \( (DR_a^t) \) that is (i) not offset by lower expected future returns \( DR_a^\infty \) and (ii) not absorbed by an increase in \( (c) \), all else being equal, has to be offset by a decrease in human wealth returns, hence the negative correlation. A 1% increase in \( DR_a^t \) has to be offset by a \( \frac{1 - \nu}{\sigma\nu} \)% decrease in \( DR_y^t \). The higher the \( \sigma \) and the higher the \( \nu \), the lower the offsetting change in \( DR_y^t \) that is required.

4.2.1.4 Volatility of human wealth returns. For the benchmark parameters and firm value returns, innovations to current and future human wealth returns \( Std(DR_y^t) \) and \( Std(DR_y^\infty) \) are 9% and 11%, only 60% and 75% as volatile as the innovations to current and future financial asset returns. As we show below, the volatility of human wealth returns further declines for larger \( EIS \).

4.2.1.5 Time-variation in the wealth shares. The human wealth share in Model V is more than twice as volatile as the labor income share. Time-variation in the human wealth share allows the model to match the moments of consumption with human wealth returns that are 20% less volatile than in the case of constant wealth shares. As a result, the market return processes are much less volatile as well. Intuitively, when the dividend yield on human wealth increases relative to the dividend yield on financial wealth, future returns on human wealth are predicted to be higher than future returns on financial wealth. This is counteracted by the lower human wealth share because \( \nu_t \) decreases in \( dp_y^t - dp_a^t \). Time-variation in the human wealth share thus reduces the volatility of the market return, and, hence, of consumption.

4.2.1.6 Market discount rate news. Innovations in the current market return are \( DR_m^t = (1 - \nu_t)DR_a^t + \nu_t DR_y^t \) and news in future market returns are given by \( DR_m^\infty = (1 - \nu_t)DR_a^\infty + \nu_t DR_y^\infty \) (see Equation 10). Table 5 displays the moments of the market return. The left panel
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Panel B, and column is Model V VAR residuals. The standard errors in \(\sigma\) are generated by a wild bootstrap (robust to heteroscedasticity).

Good news in financial markets is bad news for the current market return. We start with the firm value results. Good news in financial markets is bad news for the current market return. Panel A uses firm value returns. Panel B uses stock returns. All results are for the full sample 1947–2004 (annual data). In each panel, the first column is Model I, with \(C' = \left(\frac{\nu}{\rho}A + (1 - \rho)\nu\right)^\nu\). The second column is Model II with \(C' = 0\), and the third column is Model IV with \(C' = \nu^2\). The last column is Model V with \(C\) chosen to minimize the distance between the model-implied and actual consumption news standard deviation and correlation. Computations are done for \(\pi = .7761\) in panel A and \(\pi = .7923\) in panel B, and \(\sigma = .28\). The standard errors in ( ) are generated by bootstrapping with replacement from the VAR residuals. The standard errors in [ ] are generated by a wild bootstrap (robust to heteroscedasticity).

shows firm value results, the right panel shows stock return results. We start with the firm value results. Good news in financial markets is bad news for the current market return. \(\text{Corr}(DR^m_t, DR^m_{t-1}) < 0\), simply because \(\text{Corr}(DR^m_t, DR^m_t) < 0\) and human wealth represents 77.6% of the market portfolio on average. The market return is strongly mean reverting: \(\text{Corr}(DR^m_t, DR^m_{t-1}) = -.97\). When \(\sigma < 1\), returns must display strong (multivariate) mean reversion to lower consumption volatility and its correlation with the market return. Since there is not enough mean reversion in financial returns, our reverse-engineered discount rates for human wealth create more mean reversion in the market return. In addition, market returns are less volatile than financial asset returns because human wealth returns are less volatile and because they are
Table 6
Discount rate news—Model V—sensitivity to EIS.

<table>
<thead>
<tr>
<th>EIS</th>
<th>.12</th>
<th>.28</th>
<th>.5</th>
<th>1</th>
<th>1.5</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Panel A: Firm value returns</td>
<td>Panel B: Stock returns</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Human wealth discount rate news</td>
<td>Future human wealth discount rate news</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Std(DR&lt;sub&gt;∞&lt;/sub&gt;)</td>
<td>.159</td>
<td>.108</td>
<td>.077</td>
<td>.056</td>
</tr>
<tr>
<td></td>
<td>Corr(DR&lt;sup&lt;y&lt;/sup&gt; t , DR&lt;sup&lt;a&lt;/sup&gt; t )</td>
<td>.755</td>
<td>.843</td>
<td>.877</td>
<td>.894</td>
</tr>
<tr>
<td></td>
<td>Corr(DR&lt;sup&lt;y&lt;/sup&gt; t , DR&lt;sup&lt;∞&lt;/sup&gt; t )</td>
<td>.477</td>
<td>-.627</td>
<td>-.741</td>
<td>-.852</td>
</tr>
<tr>
<td></td>
<td>Std(DR&lt;sub&gt;y&lt;/sub&gt;)</td>
<td>.146</td>
<td>.092</td>
<td>.060</td>
<td>.039</td>
</tr>
<tr>
<td></td>
<td>Corr(DR&lt;sup&lt;y&lt;/sup&gt; t , DR&lt;sup&lt;a&lt;/sup&gt; t )</td>
<td>-.731</td>
<td>-.846</td>
<td>-.913</td>
<td>-.964</td>
</tr>
<tr>
<td></td>
<td>Corr(DR&lt;sup&lt;y&lt;/sup&gt; t , DR&lt;sup&lt;∞&lt;/sup&gt; t )</td>
<td>.370</td>
<td>.552</td>
<td>.674</td>
<td>.804</td>
</tr>
<tr>
<td></td>
<td>Std(DR&lt;sub&gt;m&lt;/sub&gt;)</td>
<td>.096</td>
<td>.049</td>
<td>.060</td>
<td>.039</td>
</tr>
<tr>
<td></td>
<td>Corr(DR&lt;sup&lt;m&lt;/sup&gt; t , DR&lt;sup&lt;a&lt;/sup&gt; t )</td>
<td>-.580</td>
<td>-.620</td>
<td>-.501</td>
<td>.207</td>
</tr>
<tr>
<td></td>
<td>Corr(DR&lt;sup&lt;m&lt;/sup&gt; t , DR&lt;sup&lt;∞&lt;/sup&gt; t )</td>
<td>.971</td>
<td>.928</td>
<td>.776</td>
<td>-.023</td>
</tr>
<tr>
<td></td>
<td>Std(DR&lt;sub&gt;c&lt;/sub&gt;)</td>
<td>.018</td>
<td>.012</td>
<td>.009</td>
<td>.011</td>
</tr>
<tr>
<td></td>
<td>Corr(c, DR&lt;sup&lt;a&lt;/sup&gt; t )</td>
<td>.207</td>
<td>.207</td>
<td>.207</td>
<td>.207</td>
</tr>
</tbody>
</table>

Panel A uses firm value returns. Panel B uses stock returns. All results are for the full sample 1947–2004 (annual data). All results are for Model V with $C$ chosen to minimize the distance between the model-implied and actual consumption news standard deviation and correlation. Computations are done for $\nu = 0.7761$ in panel A and $\nu = 0.7923$ in panel B.

negatively correlated with human wealth returns. These two forces combine to match the consumption moments.\(^{15}\)

4.2.1.7 Varying the EIS. We investigate the sensitivity of the results to the choice of the EIS parameter $\sigma$ in Table 6. Reading across the columns, for each of the calibrations we get (i) strong negative correlations between news about current and future financial and human wealth returns, as well as (ii) high and positive correlations between current financial and

\(^{15}\) Tables 4 and 5 in the separate appendix contain the results for quarterly data and for the long annual sample of stock market data. They correspond to Tables 4 and 5 in the main text. The results are very similar.
future human wealth discount rates. Good news about current financial asset returns raises risk premia on future human wealth returns and good news about current human wealth returns increases future risk premia on financial assets. These features, which are present for all values of the EIS, enable Model V to match the smooth consumption series and its low correlation with financial asset returns (last two rows).

The volatility of human wealth returns decreases in $\sigma$. What also changes across the columns are the properties of the market return. When the agent is myopic ($\sigma = 1$), we know that consumption responds one-for-one to innovations in the market return: as volatile as and perfectly correlated with the market. In the more-than-log case ($\sigma = 1.5$ in column 5), the market return must display mean aversion $\text{Corr}(DR^T, DR^\infty) > 0$ to match the consumption moments. The algorithm increases this correlation as the EIS increases by choosing a human wealth return process with large enough positive correlations $\text{Corr}(DR^H, DR^\infty)$ and $\text{Corr}(DR^T, DR^\infty)$ to overcome the mean reversion in financial asset returns and human wealth returns.

4.2.2 Failure of the benchmark Models II, III and IV. We trace the failure of the benchmark models back to their implications for human wealth returns. The consumption data clearly tell us that good news for current financial wealth returns is bad news for current human wealth returns. The benchmark models imply a positive correlation instead.

4.2.2.1 Model III. Because it equates expected future human wealth and financial wealth returns, Model II imposes the following four restrictions on human wealth discount rates:

\[ \text{Std}(DR^\infty) = \text{Std}(DR^\infty) \quad \text{and} \quad \text{Corr}(DR^T, DR^\infty) = \text{Corr}(DR^T, DR^\infty) \]
\[ \text{Corr}(CF^T, DR^\infty) = \text{Corr}(CF^T, DR^\infty) \quad \text{and} \quad \text{Corr}(DR^T, DR^\infty) = 1. \]

For Model II, news about future expected returns on human capital is very volatile, as volatile as the news about financial returns (see first column of Table 4). This is one contributing factor to the high volatility of consumption. Second, mean reversion in the financial return data acts to increase the variance of consumption innovations and the correlation of financial return innovations and consumption innovations because Model II sets $\text{Corr}(DR^T, DR^\infty) = \text{Corr}(DR^T, DR^\infty)$ and the former has a negative effect on the consumption moments. Intuitively, when good news in the stock market also leads to lower future risk premia on human wealth, positive consumption responses are magnified. The assumption $\text{Corr}(CF^T, DR^\infty) = \text{Corr}(CF^T, DR^\infty)$ similarly increases $\text{Std}(c)$ and
\( \text{Corr}(c, DR^a_t) \). The only assumption that helps to reduce the two moments is \( \text{Corr}(DR^a_\infty, DR^y_\infty) = 1 \) (when \( \sigma < 1 \)). The net result is that aggregate consumption innovations in Model II are much too volatile (by a factor of 6.1 in panel A and 6.4 in panel B) and much too highly correlated with return innovations (by a factor of 4.5 in panel A and 3.6 in panel B).

### 4.2.2.2 Model III

We expect Model III to do better because it assumes a constant discount rate for human capital, which implies that the future DR term is set to zero:

\[
\begin{align*}
\text{Std}(DR^a_\infty) &= \text{Corr}(DR^a_t, DR^a_\infty) = \text{Corr}(CF^a_\infty, DR^a_\infty) \\
&= \text{Corr}(DR^a_t, DR^a_\infty) = 0.
\end{align*}
\]

This helps lower the variance and correlation moment compared to Model II. Indeed, the standard deviation of consumption is 2.8\% per annum and the correlation moment is .825, lower than in Model II but still far away from the data (second column of Table 4). When we use stock returns instead of the returns on firm value (panel B), the predicted correlation of innovations in consumption decreases to .636, because stock market returns display more mean reversion. The variance of consumption news is still off by a factor of 4, and the correlation by a factor of 2.7.

### 4.2.2.3 Model IV

Of our three benchmark models, only Model IV comes close to delivering the same correlation pattern for \( DR^a_\infty \) as Model V: \( \text{Corr}(DR^a_t, DR^a_\infty) \) and \( \text{Corr}(DR^a_\infty, CF^a_\infty) \) are positive and \( \text{Corr}(DR^a_t, DR^y_\infty) \) is negative. For firm value data, the correlation between current financial and human wealth returns is still positive, but smaller than in Models II & III. For stock returns, Model IV even delivers a weakly negative correlation of −.07, but it is not nearly as large as the −.68 for Model V. The reason that Model IV is somewhat better lies in its assumption that news about future human wealth returns (\( DR^a_\infty \)) equals news about future labor income growth (\( CF^a_\infty \)). In the data, news about future labor income growth is not very volatile, especially compared to news about future financial asset returns. Also, \( \text{Corr}(DR^a_\infty, DR^y_t) > 0 \) helps lower the volatility and correlation of consumption innovations when the EIS is smaller than one. In the data, \( \text{Corr}(CF^a_\infty, DR^y_t) > 0 \). However, these effects are too small to substantially improve on Model III.

It follows readily that in the three benchmark models, innovations in the market return are positively correlated with innovations in financial asset returns and human wealth returns (rows 2 and 3 in Table 5). This stands in contrast to our findings for Model V.
4.2.2.4 Parameter robustness. These results are robust to plausible changes in parameter values. Figure 3 plots the model-implied standard deviation of consumption innovations and the correlation of consumption innovations with innovations in financial market returns against the EIS on the x-axis. The labor share $\pi$ is .776, and the results are for annual firm value data. None of the models comes close to matching the variance and correlation, even for very low EIS. Nonetheless, Model III and IV are closer to matching the standard deviation than Model II for a much wider range of $\sigma$. Little progress is made in matching the correlation.

4.3 Asset pricing implications

By substituting the expression for the covariance between current consumption and market return innovations back into the consumption Euler equation, Campbell (1996) derives an asset pricing formula without consumption:

$$E(t) r_{t+1}^f - r_t^f + \frac{1}{2} \text{Var}(D R_t^m) = \gamma \text{Cov}(D R_t^m, D R_t^a)$$

$$+ (\gamma - 1) \text{Cov}(D R_t^a, D R_t^m).$$

This figure plots the standard deviation of consumption innovations ($\text{Std}(c)$) and the correlation with return innovations ($\text{Corr}(c, D R_t^m)$) against the EIS. The results are for 1947–2004 (annual data). The broad firm value measure was used. The average labor (wealth) share $\pi = \pi$ is .7781. The EIS is .28. Model II sets $C' = \frac{1}{2} \left( e' \rho A + (1 - \rho) e' \right)$, Model III sets $C' = 0$, and Model IV sets $C' = e' \rho A$. Model V chooses $C'$ to minimize the distance between the model-implied and actual consumption news standard deviation and correlation.
The first part of the equity risk premium is the myopic component, governed by the correlation between news about the market return and financial asset returns. The second part is the hedging component of Merton (1973), governed by the correlation between news about the future market returns and the current financial market return. In the consumption-consistent model, the hedging risk premium is positive for all values of the $EIS$, but the myopic risk premium is negative, at least for low $EIS$. In Models I–IV, the pattern is exactly the opposite: the hedging risk premium is negative and the myopic risk premium is positive. This difference underscores the importance of measuring the market risk premium correctly. We now investigate the ability of each model to generate an equity premium of the magnitude observed in the data. The average log excess return in our 1947–2004 sample is 5.90% for stocks and 7.13% for the broader firm value measure.

4.3.0.5 Benchmark models. The upper panel of Table 7 shows that the benchmark models cannot match the equity premium, even for values of $\gamma$ as high as 50. In Models II–IV, there is a race between an offsetting myopic and hedging effect. To illustrate this, we restate the equity premium as $\text{Std}(DR^a_t) \times [\gamma \text{Std}(DR^m_t) \text{Corr}(DR^a_t, DR^m_t) + (\gamma - 1)\text{Std}(DR^m_\infty) \text{Corr}(DR^a_t, DR^m_\infty)]$. The first, myopic component is large and positive: $\text{Corr}(DR^a_t, DR^m_t)$ is consistently around 0.9 for Models II–IV (see row 2 of Table 5). The $\text{Std}(DR^a_t)$ is larger for Model II (around 15%) than for the other models (less than 5%). The second, hedging component is always negative for Models II–IV: $\text{Corr}(DR^a_t, DR^m_\infty)$ is consistently around $-0.8$ (row 5 of Table 5). This is not surprising because innovations in market returns behave largely like innovations in financial returns. The negative hedging risk premium thus mostly reflects the strong mean reversion in financial returns. Mean reversion makes financial asset returns less risky in the long run in the benchmark models. In the case of stock returns, this effect is strong enough to deliver negative risk premia for Models II and III (see Panel B). As a result, in addition to producing consumption that is at odds with consumption in the data, these benchmark models cannot match the moments of the excess returns in the data, even for high $\gamma$.

4.3.0.6 Model V. In Model V, the only consumption-consistent model, the exact opposite happens. The lower panel of Table 7 reports the equity

---

This effect is most obvious in the case with only financial wealth (Model I). In this model, the equity premium is $E(r^a_{t+1} - r^f_t) = (\gamma - 1/2)\text{Var}(DR^a_t) + (\gamma - 1)\text{Cov}(DR^a_t, DR^m_\infty)$. From the first column of Table 3 (quarterly firm value returns), we know that the myopic risk premium equals $(\gamma - 1/2) \times 1.93\%$, while the hedging risk premium equals $(\gamma - 1) \times -1.97\%$. The risk premium decreases in $\gamma$ and peaks at 1% for $\gamma = 0$.
both stock returns and firm value returns: given within [], are generated by a wild bootstrap (robust to heteroscedasticity). All results are for the full sample 1947–2004 of annual data. The relative risk aversion of the hedging risk premium is always positive, regardless of the EIS because the reverse-engineered market return depends on the EIS. First, the hedging risk premium is always positive, regardless of the EIS, for both stock returns and firm value returns: \( \text{Corr}(\text{DR}_t^m, \text{DR}_t^n) > 0 \), between .34 and .65 (row 12 in Table 6). Second, the myopic risk premium is negative for low EIS and positive for high EIS: \( \text{Corr}(\text{DR}_t^n, \text{DR}_t^m) \) switches signs (row 9 in Table 6). For low

<table>
<thead>
<tr>
<th>( \gamma )</th>
<th>Panel A: Firm value returns</th>
<th>Panel B: Stock returns</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>II</td>
<td>III</td>
</tr>
<tr>
<td>10</td>
<td>0.017</td>
<td>0.001</td>
</tr>
<tr>
<td>&amp; [0.007] &amp; [0.005] &amp; [0.009] &amp; [0.006] &amp; [0.008] &amp; [0.011]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>20</td>
<td>0.024</td>
<td>0.006</td>
</tr>
<tr>
<td>&amp; [0.012] &amp; [0.010] &amp; [0.019] &amp; [0.012] &amp; [0.017] &amp; [0.023]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>30</td>
<td>0.032</td>
<td>0.010</td>
</tr>
<tr>
<td>&amp; [0.017] &amp; [0.016] &amp; [0.029] &amp; [0.017] &amp; [0.025] &amp; [0.035]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>40</td>
<td>0.040</td>
<td>0.015</td>
</tr>
<tr>
<td>&amp; [0.023] &amp; [0.021] &amp; [0.038] &amp; [0.023] &amp; [0.034] &amp; [0.047]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>50</td>
<td>0.048</td>
<td>0.020</td>
</tr>
<tr>
<td>&amp; [0.029] &amp; [0.026] &amp; [0.048] &amp; [0.029] &amp; [0.042] &amp; [0.059]</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

This table reports \( \text{Eq}_t^V/\sqrt{\text{Var}(DR_t^m)} + \gamma \text{Corr}(DR_t^m, DR_t^n) + (\gamma - 1) \text{Corr}(DR_t^m, DR_t^n) \). Panel A uses firm value returns. Panel B uses stock returns. All results are for the full sample 1947–2004 of annual data. The upper panel shows the implied equity premium for Models II–IV. These only depend on the coefficient of relative risk aversion \( \gamma \). The lower panel shows the equity premium for Model V. It depends on \( \gamma \) and the EIS \( \nu \). Computations are done for \( \nu = 0.7761 \) in panel A and \( \nu = 0.7923 \) in panel B. The standard errors, given within [], are generated by a wild bootstrap (robust to heteroscedasticity).
EIS, the first effect dominates the second effect and the risk premia are large. For example, when \( \sigma = 0.12 \) and \( \gamma \) is 20, the risk premium is 8.1\% and close to the data. The standard errors are large: small variations in the estimated moments can change the sign of the risk premium because the components have an offsetting effect. For large EIS, both the hedging and the myopic component of the risk premium are positive. The resulting risk premium is large and more precisely estimated because the hedging and myopic risk premium have the same sign. The risk premium is 6.3\% when \( \sigma = 1.5 \) and \( \gamma \) is 20. The relation between EIS and the risk premium is non-monotonic. For \( \sigma < 1 \), as the EIS increases, the risk premium initially decreases, because the \( Std(DR_{m}) \) decreases. For \( \sigma = 0.5 \), the risk premium even becomes negative. However, for \( \sigma > 1 \), the risk premium increases as the EIS increases. In sum, Model V can match the risk premium, either for a small or a large EIS. Using firm value (stock) returns, we match the equity premium for \( \gamma = 18 \) (\( \gamma = 12 \)) when \( \sigma = 0.12 \). We also match the equity premium for \( \gamma = 22 \) (\( \gamma = 19 \)) when \( \sigma = 1.5 \).

5. Other Explanations

We attribute the component of aggregate consumption growth that is not accounted for by financial asset returns to human wealth returns. Other labels come to mind for this residual. We consider four alternatives, and we find that these are unlikely to resolve the consumption volatility and correlation puzzles.

First, if the agent’s preferences display external habit formation as in Campbell and Cochrane (1999), the volatility and the correlation puzzles cannot be resolved, unless through heteroscedasticity in the market return. We test for this possibility below. Second, the omission of housing wealth may lead to the erroneous interpretation of the residual as a human wealth return. However, when we include housing wealth in the portfolio of the investor, the residual has the same properties as in the model without housing wealth.

5.1 Heteroscedastic market returns

We have abstracted from time-variation in the joint distribution of consumption growth and returns. Recently, Duffee (2005) reports finding some evidence of time-variation in the covariance between stock returns and consumption growth. We denote the conditional variance term by \( \mu_{m}^{t} \). This adds a third source of consumption innovations [Equation (38)]

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17 See separate appendix Section B.1 for a formal proof and a detailed discussion of the habit model.

18 See separate appendix Section B.2 for a model with housing. The results from redoing the entire estimation exercise with housing wealth are similar to the ones reported here.
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in Campbell (1993)], which reflects the influence of changing risk on the household’s saving decisions:

\[(c)_t = DR^m_t + (1 - \sigma)DR^\infty_t - (E_t - E_{t-1}) \sum_{j=1}^{\infty} \rho^j \mu^m_{t+j},\]

where \(\mu^m_t = \sigma \log \beta + 0.5(1 - \sigma)^2 \frac{2}{\sigma} Var_t[DR^m_{t,\infty}].\) This last term drops out if either \(\gamma\) or \(\sigma\) is one. We refer to this last term as news about future variances, \(FV^\infty.\) If this time-variation plays a role, our consumption growth accounting residual should predict the future variance of stock market returns, and/or the future variance of consumption and/or the conditional covariance between the two. We check whether the residual that comes out of our model with time-varying human wealth shares predicts \(Var_t[DR^m_{t,\infty}],\) and we find that it does not.20

5.2 Heterogeneity

Fourth, we consider heterogeneity across households, and we argue that reasonable specifications of heterogeneity only make the puzzle worse. When households have the same EIS, aggregation reproduces exactly Equation (4) for aggregate consumption innovations under fairly mild conditions, and all the previous results go through trivially. However, if household wealth and the EIS are positively correlated, then the aggregate EIS that shows up in the aggregate consumption innovation expression exceeds the average EIS across households. In fact, Vissing-Jorgensen (2002) reports evidence of a higher EIS for wealthier stock- and bondholders. A higher aggregate EIS worsens the consumption volatility and correlation puzzles.

6. Conclusion

From the perspective of a standard neoclassical growth model, the volatility of consumption innovations relative to that of financial return innovations is too small, as is the correlation of consumption and financial return innovations, even if the representative agent is very reluctant to substitute consumption over time. One possible resolution of these puzzles lies in the

19 Note that because \(\theta > 0\) when \(\gamma > 1\) and \(\sigma < 1,\) there is a positive relationship between the conditional variance of news about current and future market returns \(DR^m_{t,\infty}\) and \(FV^\infty.\)

20 First, we construct \(FV^\infty \equiv -(c)_t + DR^m_t + (1 - \sigma)DR^\infty_t\) for Models I, II, III, and IV. Second, to calculate time-varying variances of news about current and future market returns, we assume that \(DR^m_{t,\infty}\) follows an AR(1) and we estimate the innovations. Finally, we regress the squared residuals from the AR(1) \(h = 1, 2, \ldots, H\) periods ahead on the current residual \(h.\)

21 The conditions are described in Section B.3 of the separate appendix.
measurement of human wealth returns. If this is the route one chooses, then the returns on human wealth need to be negatively correlated with returns on financial assets in order to generate a consumption process that is consistent with the data. This result reflects both negative correlation in news about the future discount rates and cash flows of financial and human wealth. Standard production functions in business cycle models, such as the Cobb-Douglas, imply a nearly perfectly correlated return on human and financial wealth [Baxter and Jermann (1997)]. Our results suggest that this is counterfactual and that we may need to think of different technologies. Models with time-varying factor elasticities, such as the one of Young (2004), may allow for a better description of the data.

Appendix A:

A.1 Bootstrap standard errors

We conduct two bootstrap exercises to compute standard errors on the moments of the data and the moments of the various models. In the exercise, we assume that the VAR innovations are i.i.d. We draw with replacement entire rows of the innovation matrix so as to preserve the cross-sectional correlation between the innovations in the various series ($1 \times N$). Each bootstrap simulation is of the same length as the data ($T$). Using these $T \times N$ matrices of VAR innovations $\{\tilde{e}_t\}$, we recursively build up the time series of the $N$ VAR elements. We then reestimate the companion matrix $A$ and error covariance matrix $\Sigma$ and form the moments of interest. In the second exercise, we use the “wild bootstrap” procedure of Gonçalves and Kilian (2003). It deals with conditional heteroscedasticity of an unknown form in auto-regressions. The procedure is a simple extension of the standard bootstrap: the actual innovations $\{\tilde{e}_t\}$ are multiplied by an i.i.d. normally distributed random variable $\{\eta_t\}$ with mean zero and standard deviation 1. Again, to preserve cross-sectional correlation, we multiply the entire row $\tilde{e}_t$ by the same scalar random variable $\eta_t$. Each bootstrap iteration represents a different time series for $\{\eta_t\}$. Once the new residuals $\{\tilde{e}_t \eta_t\}$ are constructed, we recursively build up the time series from the VAR. We then reestimate the companion and error covariance matrix and form the moments of interest.

A.2 Imposing co-integration

This appendix explains how to impose co-integration between consumption, financial wealth and labor income. Much of it follows Campbell (1993) and Lettau and Ludvigson (2001).

A.2.0.7 Deriving the co-integration relationship. Define total wealth (in levels) to be $M_t = A_t + H_t$; it consists of financial wealth $A$ and human wealth $H$. Denote log variables by lower case letters. Log wealth can be written as $m_t = (1 - \nu) a_t + \nu h_t$, where $\nu$ is the average human wealth share. Likewise, returns on the market portfolio are a linear combination of returns on financial and human wealth: $r^m_t = (1 - \tau) r^a_t + \tau r^y_{t+1}.$

We start by linearizing the budget constraint around the mean log consumption–wealth ratio. This is an approximation that we take to be the true model:

$$\Delta m_{t+1} = r^m_{t+1} + k + \left(1 - \frac{1}{m_0}\right) (c_t - m_t)$$ (A1)
where \( k = \log(\rho) - \left( 1 - \frac{1}{\rho} \right) c - m \) and \( \rho = 1 - \exp(c - m) \). The same equation holds for the growth rate of financial wealth \( \Delta_d t \):

\[
\Delta dt_{t+1} = r_{t+1}^f + k + \left( 1 - \frac{1}{\rho} \right) (d_t - a_t)
\]

(A2)

where \( d_t \) denotes financial income, and for human wealth changes \( \Delta h_{t+1} \):

\[
\Delta h_{t+1} = r_{t+1}^h + k + \left( 1 - \frac{1}{\rho} \right) (y_t - h_t)
\]

(A3)

We have assumed that the linearization constants \( k \) and \( \rho \) are the same for the three sources of wealth. It follows from the expressions for \( \Delta m_{t+1} \), \( \Delta a_{t+1} \), and \( \Delta h_{t+1} \), and from \( \Delta m_{t+1} = (1 - \tau) \Delta a_{t+1} + \tau \Delta h_{t+1} \) that \( c_t - m_t = (1 - \tau)(d_t - a_t) + \tau(y_t - h_t) \), i.e., the log dividend–price ratios on the wealth components are linearly related.

The next step is to iterate forward each of these three equations. For example, we substitute the identity \( \Delta m_{t+1} = \Delta c_{t+1} + (c_t - m_t) - (c_{t+1} - m_{t+1}) \) into Equation (A1) and impose \( \lim_{t \to \infty} \rho^j (c_{t+j} - m_{t+j}) = 0 \) to get an expression for the consumption–wealth ratio, the dividend–price ratio on total wealth:

\[
c_t - m_t = \sum_{j=1}^{\infty} \rho^j (r_{t+j}^m - \Delta c_{t+j}) + \frac{\rho}{1-\rho} k
\]

Likewise, we obtain the dividend–price ratio on financial wealth:

\[
d_t - a_t = \sum_{j=1}^{\infty} \rho^j (r_{t+j}^h - \Delta d_{t+j}) + \frac{\rho}{1-\rho} k
\]

and on human wealth:

\[
y_t - h_t = \sum_{j=1}^{\infty} \rho^j (r_{t+j}^h - \Delta y_{t+j}) + \frac{\rho}{1-\rho} k
\]

(A4)

The last step is to substitute the expressions for \( m \) and \( r^m \) into the expression for \( c_t - m_t \),

\[
c_t - (1 - \tau)d_t - \tau y_t = \sum_{j=1}^{\infty} \rho^j ((1 - \tau)r_{t+j}^m + \tau r_{t+j}^h - \Delta c_{t+j}) + \frac{\rho}{1-\rho} k
\]

to solve (A4) for \( h_t \) and to substitute this expression into the above equation:

\[
c_t - (1 - \tau)d_t - \tau y_t = \sum_{j=1}^{\infty} \rho^j ((1 - \tau)r_{t+j}^m + \tau \Delta y_{t+j} - \Delta c_{t+j}) + (1 - \tau) \frac{\rho}{1-\rho} k
\]

(A5)

This expression holds \textit{ex post} but also \textit{ex ante}. Imposing that \( E_t \sum_{j=1}^{\infty} \rho^j ((1 - \tau)r_{t+j}^m + \tau \Delta y_{t+j} - \Delta c_{t+j}) \) is stationary, the left-hand side must also be stationary. This is the co-integration relationship between consumption, financial wealth and labor income, or \( c_{ay} \), from Lettau and Ludvigson (2001). The same argument goes through with time-varying wealth shares. The right hand side of Equation (A5) then contains an additional component, \( \tilde{v}_t (y_t - d_t) + \sum_{j=1}^{\infty} \rho^j (\tilde{v}_{t+j} (y_{t+j} - d_{t+j}) + \tilde{v}_t \sum_{j=1}^{\infty} \rho^j (\Delta y_{t+j} - \Delta d_{t+j})) \), which must also be stationary.

\textbf{A.2.0.8 Empirical proxy for consumption-wealth ratio.} Following Lettau and Ludvigson (2001), we have estimated a vector error correction model (VECM) with...
consumption \((c_t)\), labor income \((y_t)\) and financial wealth \((a_t)\). All variables are in logs and expressed in real per capita terms. Financial wealth is either stock market wealth or firm value wealth. As part of this estimation, we retrieve the coefficients in the cointegrating vector \(\lambda c_t = c_{t-1} + \alpha - \beta_a a_t - \beta_y y_t\). We follow Lettau and Ludvigson (2001), who argue that non-durable consumption and services are only a fraction of total consumption and postulate \(\lambda c_t = \epsilon_t^{new}\). We set \(\lambda = 1.03946\) equal to the 1947–2004 sample average of the ratio of log personal consumption expenditures to log non-durable consumption (non-durables and services excluding housing services). To stay with the model, we impose the restriction that \(\beta_a + \beta_y = 1\). This follows from \(\beta_a = 1 - \pi\) and \(\beta_y = \pi\). Basically, the wealth shares must add to one. We estimate the co-integration coefficients by dynamic least squares:

\[
\lambda c_t = \alpha + \beta_a a_t + \beta_y y_t + \sum_{j=-4}^{k} b_{t-j} \Delta a_{t+j} + \sum_{j=-k}^k b_{t-j} \Delta y_{t+j}.
\]

To keep matters simple, we estimate one \(\pi\) for firm value data and one \(\pi\) for stock returns. We use the common sample 1947–2004 of quarterly data. We find strong evidence for one cointegrating vector between consumption, labor income, and financial wealth. First, when financial wealth is stock market wealth, the Johansen trace statistic is 41.58, so that the null of no co-integration is rejected at the 1% level (32 lags, 203 observations total). The point estimates (and their standard errors) are \(\hat{\beta}_a = 0.2077(0.0879)\) and \(\hat{\beta}_y = 0.7923(0.0879)\). These estimates imply an average human wealth share of \(\pi = 0.7923\). Second, for firm wealth returns, the trace statistic is 45.79, also significant at the 1% level (32 lags, 202 observations total). Our estimates now are \(\hat{\beta}_a = 0.2239(0.0528)\) and \(\hat{\beta}_y = 0.7761(0.0528)\). These imply an average human wealth share of \(\pi = 0.7761\). So, our measure of cay is \(c_{ay_i} = \lambda c_{i-1} - (1 - \pi) a_i - \pi y_i\). We form the \(cay\)-augmented VAR, as in Equation (8) in the main text. We then redefine the companion matrix \(A\) to be the \(8 \times 8\) companion matrix of the augmented VAR system and \(\epsilon\) to be the augmented \(8 \times T\) innovation vector. We note that its covariance matrix is singular, because the \(7^{th}\) element is a linear combination of three elements of the original \(7 \times 1\) innovation vector.

### A.3 Time-varying wealth share

This appendix provides more detail on how to deal with time-varying wealth shares. Because \(dp_t^a\) is a function of the entire state space, so is \(v_t\). \(v_t\) is not a linear, but a logistic function of the state. We use a linear specification \(v_t = v_1 - \pi = D' z_t\) and we pin down \(D\) \((N \times 1)\) using a first order Taylor approximation. Let \(s_t\) be the labor income share with mean \(\pi\) and \(w_t = dp_t^a - dp_t\) with mean zero. (The mean of \(w_t\) must be zero to be able to use the same linearization constant \(\rho\) for human wealth and financial wealth.) We can linearize the logistic function for the human wealth share \(v_t\) from Equation (15) using a first order Taylor approximation around \((s_t = \pi, w_t = 0)\). We obtain:

\[
v_t(s_t, w_t) \approx v_t(\pi, 0) + \left[ \frac{\partial v_t}{\partial s_t} \right]_{s_t = \pi, w_t = 0} (s_t - \pi) + \left[ \frac{\partial v_t}{\partial w_t} \right]_{s_t = \pi, w_t = 0} (w_t),
\]

\[
\approx \pi + (s_t - \pi - (\pi (1 - \pi)) w_t
\]

\[
= s_t - \pi (1 - \pi) dp_t + \pi (1 - \pi) dp_t
\]

(A6)

The average human wealth share is the average labor income share: \(\bar{\pi} = \pi\). If \(dp_t\) is the third element of the VAR, \(dp_t = \epsilon_t z_t\), and \(s_t - \pi\) the sixth, and if \(dp_t^a = B' z_t\), then we can solve for \(D\) from Equation (A6) and \(v_t = D' z_t\):

\[
D = \epsilon_t - \pi (1 - \pi) B + \pi (1 - \pi) e_t.
\]

(A7)

With the portfolio weights \(v_t\), we can construct consumption innovations accordingly. The difficulty is to calculate the terms \(D R^{a,a}\) and \(D R^{a,s}\) in Equation (17). We compute the

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innovations $\Delta R^{w,a}$ and $\Delta R^{w,y}$ using value function iteration. First, we define the expected weighted future asset returns $\Delta R^{w,a}$ as

$$\Delta R^{w,a}(z_t) = E_t \sum_{j=1}^{\infty} \rho^j \tilde{\nu}_{t-1+j} r_{t+j}^a$$

$$= \tilde{\nu}_t \rho e_t^a + E_t \sum_{j=2}^{\infty} \tilde{\nu}_{t-1+j} \rho^j E_t \tilde{\nu}_{t-1+j} r_{t+j}^a$$

$$= \tilde{\nu}_t \rho e_t^a + \rho E_t \sum_{j=2}^{\infty} \tilde{\nu}_{t-1+j} \rho^{j-1} E_t \tilde{\nu}_{t-1+j} r_{t+j}^a$$

$$= \frac{\tilde{\nu}_t}{\epsilon} \frac{\rho}{\epsilon} C^a \epsilon_t A \tilde{\nu}_{t-1} \rho e_t^a + \rho E_t \tilde{\nu}_{t-1} \rho^{j-1} C^a \epsilon_t \tilde{\nu}_{t-1+j} r_{t+j}^a$$

(A8)

and similarly for $\Delta R^{w,y}$. In a second step, we exploit the recursive structure of $\Delta R^{w,a}$ and $\Delta R^{w,y}$ to show that $\Delta R^{w,a}$ can be stated as a quadratic function of the state:

$$\Delta R^{w,a}(z_t) = \zeta^r P z_t + d$$

where $P$ solves a matrix Sylvester equation, whose fixed point is found by iterating on

$$P_{j+1} = R + \rho A' P_j A, \quad (A9)$$

starting from $P_0 = 0$, and $R = \rho D e_t A$. The constant $d$ equals $\frac{1}{\epsilon} \frac{\rho}{\epsilon} tr(P \Sigma)$. We are interested in:

$$\Delta R^{w,a}(z_t) = (E_t - E_{t-1}) \Delta R^{w,a}(z_t) = (E_t - E_{t-1})[\zeta^r P z_t + d]$$

$$= \epsilon_t^a P e_t - E_{t-1}[\epsilon_t^a P e_t]$$

$$= \epsilon_t^a P e_t - \sum_{i=1}^{N} \sum_{j=1}^{N} \Sigma_{ij} P_{ij}$$

which turns out to be a simple quadratic function of the VAR shocks and the matrix $P$. In the same manner we calculate $\Delta R^{w,y}$, replacing $R$ in Equation (A9) by $\rho D C^y$. $C$ takes on different values for the three canonical models.

Finally, we can compute innovations to the total market return $\Delta R^m = \epsilon_t^m - E_{t-1}[\epsilon_t^m]$:

$$\Delta R^m = (\tilde{\nu}_{t-1} + \nabla (\epsilon_t^m - E_{t-1}[\epsilon_t^m]) + (1 - \tilde{\nu}_{t-1} - \nabla \epsilon_t^m - E_{t-1}[\epsilon_t^m])$$

$$= (\tilde{\nu}_{t-1} + \nabla) \Delta R^m + (1 - \tilde{\nu}_{t-1} - \nabla) \Delta R^m$$

$$= \frac{\tilde{\nu}_{t-1} + \nabla \epsilon_t^m - \rho C^y (1 - \rho A)^{-1} + (1 - \tilde{\nu}_{t-1} - \nabla \epsilon_t^m) e_t}{\epsilon_t^m}$$

and news in future market returns $\Delta R^m = (E_t - E_{t-1}) \sum_{j=1}^{\infty} \rho^j \epsilon_{t+j}^m$:

$$\Delta R^m = (E_t - E_{t-1}) \sum_{j=1}^{\infty} \rho^j \left[ (\tilde{\nu}_{t-1+j} + \nabla \epsilon_{t+j}^m + (1 - \tilde{\nu}_{t-1+j} - \nabla) \epsilon_{t+j}^m) \right]$$

$$= \nabla N F Y e_t + \Delta R^m, + (1 - \nabla) \Delta R^m - \Delta R^m$$

$$= \rho \left[ \epsilon_t^m + (1 - \nabla) \epsilon_t^m \right] (1 - \rho A)^{-1} e_t - (\epsilon_t^m (P - Q) e_t) - q$$

where the constant $q = \sum_{i=1}^{N} \sum_{j=1}^{N} \Sigma_{ij} (P_{ij} - Q_{ij})$. 2133
A.4 Long-run restriction

The household budget constraint imposes a restriction on the long-run effect of news about market returns and consumption growth:

\[(E_{t+1} - E_t) \sum_{j=0}^{\infty} \rho^j \Delta c_{t+1+j} = (E_{t+1} - E_t) \sum_{j=0}^{\infty} \rho^j \Delta c_{t+1+j},\]

or in our notation: \(DR_t^m = CF_{t,\infty}\), where \(CF_{t,\infty}\) means innovations in current and future consumption growth, the cash flows on the market portfolio.

As pointed out in Hansen et al. (1991), this restriction cannot be satisfied for the models with constant wealth shares. In the constant wealth share case, we obtain:

\[DR_t^m + DR_t^e = (1 - \tau)DR_t^e + (1 - \tau)DR_t^w + \tau DR_t^e + \tau DR_t^w = (1 - \tau)C_{t,\infty}^F + \tau C_{t,\infty}^C,\]

where we have used that \(DR_t^e = CF_{t,\infty}^e - DR_t^w\) and \(CF_{t,\infty}^w = DR_t^e + DR_t^w\). Therefore,

\[(1 - \tau)C_{t,\infty}^F + \tau C_{t,\infty}^C = CF_{t,\infty}^E. \quad (A10)\]

Since consumption growth is taken from the data (it is the seventh element in the VAR), the long-run response of consumption growth can be computed as:

\[CF_{t,\infty}^c = (E_t - E_{t-1}) \sum_{j=0}^{\infty} \rho^j \Delta c_{t+1+j} = e_t'(I - \rho A)^{-1} \xi_t.\]

Likewise, \(CF_{t,\infty}^e = \epsilon_t'(I - \rho A)^{-1} \xi_t\) and \(CF_{t,\infty}^w = [\epsilon_t'(I - \rho A)^{-1} + \epsilon_t'(1 - \rho)(I - \rho A)^{-1}] \xi_t\). The equality between news about long-run returns and consumption growth should hold for all \(\xi_t\). Thus, Equation (A10) imposes that

\[(1 - \tau)[\epsilon_t'(I - \rho A)^{-1} + \epsilon_t'(1 - \rho)(I - \rho A)^{-1}] + \tau \epsilon_t' = e_t'(I - \rho A)^{-1},\]

or post-multiplying by \((I - \rho A)\) shows this implies \((1 - \tau)(\epsilon_t' + \epsilon_t'(1 - \rho)) + \tau \epsilon_t' = e_t'\), which cannot be satisfied because the vector on the right hand side has zeros in all entries but the seventh, while the left-hand side has non-zero elements in the first, second, and third entries. So, the linearity of the VAR implies that the budget constraint cannot be satisfied exactly for all innovations.

The same restriction is also violated for model-implied consumption innovations. Recall the optimal consumption rule that follows from the Euler equation:

\[c_t = (1 - \tau)DR_t^e + \tau CF_{t,\infty}^c - \sigma \tau DR_t^e + (1 - \sigma)(1 - \tau)DR_t^w,\]

\[= \left[(1 - \tau)e_t' + \tau \epsilon_t'(I - \rho A)^{-1} - \sigma \tau \epsilon_t'(I - \rho A)^{-1}\right] \xi_t + (1 - \sigma)(1 - \tau)\left[\epsilon_t'(I - \rho A)^{-1} + \epsilon_t'(1 - \rho)(I - \rho A)^{-1}\right] \xi_t,\]

where we have used \(DR_t^e = C_{t,\infty}^C(I - \rho A)^{-1} \epsilon_t\). The discounted infinite sum of consumption innovations is \(CF_{t,\infty}^c = (I - \rho A)^{-1} c_t\). This needs to equal

\[\left[(1 - \tau)e_t' + \tau \epsilon_t'(I - \rho A)^{-1} + \epsilon_t'(1 - \rho)(I - \rho A)^{-1}\right] + \tau \epsilon_t'(I - \rho A)^{-1} \xi_t.\]
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This implies that the following equality must hold for all $\epsilon_t$:

$$(1 - \varpi)\epsilon_t'(I - \rho A)+\varpi \epsilon_t'-\sigma \varpi \epsilon_t'+(1 - \sigma)(1 - \varpi)[(\epsilon_t'\rho A+\epsilon_t'(1-\rho)]$$

$$=(1 - \varpi)[\epsilon_t'+\epsilon_t'(1-\rho)]+\varpi \epsilon_t',$$

which would require

$$-\sigma \varpi \epsilon_t'-\sigma(1 - \varpi)[(\epsilon_t'\rho A+\epsilon_t'(1-\rho)]$$

$$=0.$$

A.4.0.9 Long-run restriction in model with time-varying wealth shares.

Importantly, for the models with time-varying wealth shares $\nu_t$, the above argument does not work. Recall the definition of the human wealth share in terms of its mean and its deviations from the mean: $\nu_t = \overline{\nu} + \tilde{\nu}_t$. We obtain

$$DR_m^w + DR_m^\infty = (1 - \varpi - \tilde{\nu}_{t-1}) DR_a^w + (E_t - E_{t-1}) \sum_{j=1}^{\infty} \rho^j(1 - \varpi - \tilde{\nu}_{t-1-j}) r^\alpha_{t+j}$$

$$+(\varpi + \tilde{\nu}_{t-1}) DR_y^w + (E_t - E_{t-1}) \sum_{j=1}^{\infty} \rho^j(\varpi + \tilde{\nu}_{t-1-j}) r^\gamma_{t+j}$$

Using the definitions for news about weighted future financial asset returns ($DR_w^a$) and human wealth returns ($DR_w^y$) from appendix A.3, the expression for model-implied cash flow news in consumption reduces to

$$DR_m^w + DR_m^\infty = (1 - \varpi - \tilde{\nu}_{t-1}) DR_a^w + (\tilde{\nu}_{t-1} + \varpi C F_{t-\infty} - \tilde{\nu}_{t-1}) DR_a^\infty$$

$$+(1 - \varpi) DR_y^\infty - (DR_w^a - DR_w^y).$$

Now, $DR_w^a$ and $DR_w^y$ are quadratic, not linear. The problem that arose before is gone because of the nonlinearity. The condition that this expression equals the long-run consumption growth response ($CF_c$) does not simplify as before, because of the quadratic terms. In addition, model-implied consumption is nonlinear as well (see Equation 17).

References


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